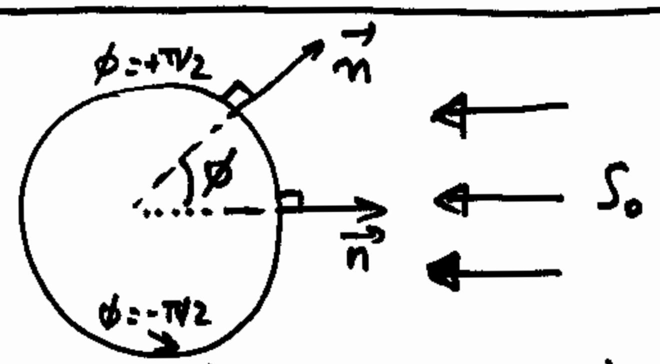
EP Problem Sheet #2 — Answers 1/4

①



at a latitude ϕ and longitude λ the projection of the solar energy flux onto the local normal ($\vec{n}$) is:

$$S_0 \cos\phi \cos\lambda$$

The infinetesimal area element at (λ, ϕ) is

$$dS = (R\cos\phi \, d\lambda) \times R\, d\phi$$

∴ energy received by Earth per unit time is

$$\int_{-\pi/2}^{+\pi/2} S_0 \cos\lambda \int_{-\pi/2}^{+\pi/2} \cos\phi \,(R^2 \cos\phi \, d\phi \, d\lambda)$$

$$= R^2 S_0 \int_{-\pi/2}^{+\pi/2} \cos\lambda \, d\lambda \int_{-\pi/2}^{+\pi/2} \cos^2\phi \, d\phi = \pi R^2 S_0$$

$\sim 175\,PW = 175{,}000\,TW!$

② $T_E \equiv \left[\frac{(1-\alpha_p) S_0}{4\sigma} \right]^{1/4}$ standard values: $S_0 = 1{,}368\ Wm^{-2}$ $\alpha_p = 0.3$

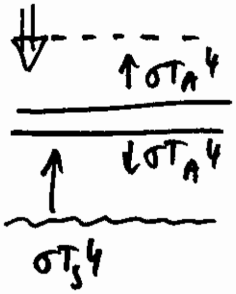
(i) with $S_0 \downarrow$ by 30% but keeping $\alpha_p = 0.3$

$$T_E \sim \left[\frac{(1-0.3)\, 910}{4 \times 5.7 \cdot 10^{-8}} \right]^{1/4} \sim 230K$$

The Earth was much warmer back then because, one believes, the atmospheric CO_2 concentration was much larger than present. The surface gained more energy through atmospheric radiation from above than from the Sun.

(ii) $\alpha_p = 0.1 \therefore T_E \sim 271K$. This is below freezing so the "Moon model" can not be used for this calculation (it leads to an inconsistency).

③



Top of Atmosphere: $\frac{(1-\alpha_p)S_0}{4} = \sigma T_A^4$

surface: $\begin{pmatrix}\text{downward}\\ \text{arrow}\end{pmatrix} = \begin{pmatrix}\text{upward}\\ \text{arrow}\end{pmatrix}$

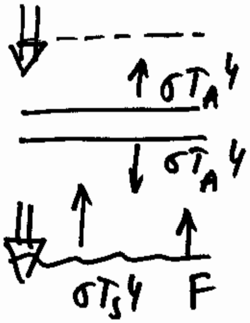
$\sigma T_A^4 = \sigma T_S^4$

$\therefore T_A = T_S = T_E$! Even with a perfect absorbing layer the surface temp. is the same as that obtained with the "Moon model"! What matters for the greenhouse effect is to absorb radiation from the surface <u>AND</u> leave un-attenuated the solar radiation.

④ (i) $\Gamma = \frac{303-255}{5} = 9.6\ K/km$ (ii) $\Gamma = \frac{288-255}{5} = 6.6\ K/km$

The first temperature profile is most likely to be unstable.

⑤



Top of Atm.

surface

$$\begin{cases} \frac{(1-\alpha_p)S_0}{4} = \sigma T_A^4 \Rightarrow T_A = T_E \\ \frac{(1-\alpha_p)S_0}{4} + \sigma T_A^4 = \sigma T_S^4 + F \end{cases}$$

From the second equation:

$$F = \frac{(1-\alpha_p)S_0}{4} + \sigma T_A^4 - \sigma T_S^4$$

using $T_A = T_E$ and $T_s = T_E + \Gamma z_E$ this becomes

$$F = \underbrace{\frac{(1-\alpha_p)S_0}{4}}_{\uparrow\ \sigma T_E^4} + \sigma T_E^4 - \sigma(T_E + \Gamma z_E)^4$$

$$= 2\sigma T_E^4 - \sigma T_E^4\Big(1 + \underbrace{\frac{\Gamma z_E}{T_E}}_{\ll 1}\Big)^4 \simeq \sigma T_E^4\Big(1 - \underbrace{\frac{4\Gamma z_E}{T_E}}_{\sim 1/2}\Big)$$

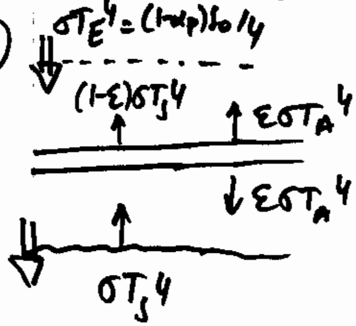
$$F \simeq \frac{1}{2}\sigma T_E^4 \sim 120\,\mathrm{W\,m^{-2}}$$

when $S_0 \uparrow$ by 10%, $T_E \uparrow$ by $(1.1)^{1/4} - 1 = 2\%$
thus $F \uparrow$ by $\{(1.02)^{1/4} \times 120 - 120\} \simeq 10\ \mathrm{W\,m^{-2}}$

⑥ $$\tau_\infty \equiv \int_0^{+\infty} k_A M_A \rho\, dz \underset{\substack{\uparrow \\ \text{uniform} \\ \text{absorber}}}{=} k_A M_A \underbrace{\int_0^{+\infty} \rho\, dz}_{\substack{\text{mass of atmosphere} \\ \text{per unit area}}}$$

$$\therefore\ k_A = \frac{\tau_\infty}{M_A \int_0^{+\infty} \rho\, dz} \simeq \frac{2}{10^{-2} \times 10^4} = 2 \cdot 10^{-2}\ \mathrm{m^2\,kg^{-1}}$$

⑦



(i) $$\begin{cases} \sigma T_E^4 = (1-\varepsilon)\sigma T_s^4 + \varepsilon\sigma T_A^4 & (1) \\ \sigma T_E^4 + \varepsilon\sigma T_A^4 = \sigma T_s^4 & (2) \end{cases}$$

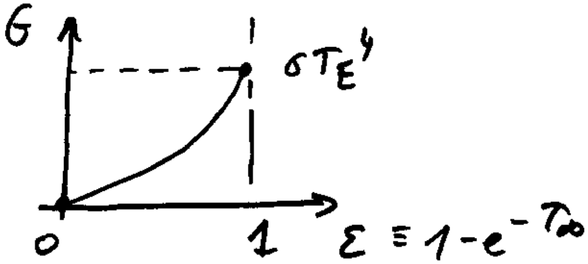
(ii) $$G = \sigma T_s^4 - [\sigma T_s^4(1-\varepsilon) + \varepsilon\sigma T_A^4]$$
$$= \varepsilon\sigma(T_s^4 - T_A^4)$$

Combining (1) & (2) we easily obtain

$$\sigma T_s^4 = \frac{2}{2-\varepsilon}\sigma T_E^4$$

$$\sigma T_A^4 = \frac{1}{2-\varepsilon}\sigma T_E^4$$

$$\therefore\ G = \varepsilon \left[\frac{2}{2-\varepsilon}\sigma T_E^4 - \frac{1}{2-\varepsilon}\sigma T_E^4 \right] = \frac{\varepsilon}{2-\varepsilon}\sigma T_E^4$$



optically "thin" = Moon model (no greenhouse effect)

$\tau_\infty = 0$

optically "thick" = Blanket model (strongest greenhouse effect)

$\tau_\infty \gg 1$