Environmental Physics - Problem Sheet 3 Answers

① (13.3) : $f_{H_2O} \equiv \Gamma \dfrac{\partial z_E}{\partial H_2O} \dfrac{\partial H_2O}{\partial T_s}$

$$= \Gamma \frac{\partial z_E}{\partial [RH\, e_{sat}]} \frac{\partial [RH\, e_{sat}]}{\partial T_s}$$

using RH = constant $\rightarrow$ $= \Gamma \dfrac{\partial z_E}{\partial e_{sat}} \left(\dfrac{d e_{sat}}{d T_s} \right) \leftarrow$ e_{sat} depends only upon T

<u>Clausius-Clapeyron</u>: $f_{H_2O} = \Gamma \dfrac{\partial z_E}{\partial e_{sat}} \dfrac{L_v}{R_v T} \dfrac{e_{sat}}{T}$

$$\therefore f_{H_2O} \simeq \frac{L_v}{R_v T} \frac{\Gamma}{T} \frac{\Delta z_E}{\Delta e_{sat}/e_{sat}}$$

$$\simeq 20 \times \frac{6.5}{288} \frac{0.1}{0.1} = 0.45$$

<u>NB</u>: note the strong increase of e_{sat} with T_s. For a 1% change in T_s ($\simeq 3K$) $\dfrac{de_{sat}}{e_{sat}} \simeq \dfrac{L_v}{R_v T} \dfrac{dT}{T} \sim 20\%$!

② (i) $dT_s = dT_E + \Gamma dz_E$ assuming fixed lapse-rate

$$= -\frac{S_0}{16\sigma T_E^3} d\alpha_p + \Gamma dz_E$$

using definition of T_E

since $d\alpha_p = (1+c)\, d\alpha_s + \alpha_s\, dc$

$$dT_s = -\frac{S_0(1+c)}{16\sigma T_E^3} d\alpha_s - \frac{S_0 \alpha_s}{16\sigma T_E^3} \frac{dc}{dT_s} dT_s + \Gamma dz_E$$

$\therefore f_{c_{neg}} = -\dfrac{S_0 \alpha_s}{16\sigma T_E^3} \underbrace{\dfrac{dc}{dT_s}}_{>0} < 0$ increasing cloud cover increases albedo so more solar radiation is reflected: negative feedback of clouds since we have assumed the increase in cloud cover is due to an increase in T_S.

(ii) $z_E = z_E(CO_2, H_2O, c, \dots)$
cloud cover

$$dz_E = \frac{\partial z_E}{\partial CO_2} dCO_2 + \frac{\partial z_E}{\partial H_2O} dH_2O + \frac{\partial z_E}{\partial c} dc + \dots$$

$$\frac{\partial z_E}{\partial c} dc = \frac{\partial z_E}{\partial c} \frac{dc}{dT_S} dT_S \quad \text{hence:}$$

$$dT_S = -\frac{S_0(1+c)}{16\sigma T_E^3} d\alpha_S + (f_{c_{neg}} + f_{c_{pos}}) dT_S + \Gamma\left(\frac{\partial z_E}{\partial CO_2} dCO_2 + \frac{\partial z_E}{\partial H_2O} dH_2O\right)$$

where $f_{c_{pos}} \equiv \Gamma \underbrace{\frac{\partial z_E}{\partial c}}_{>0} \underbrace{\frac{dc}{dT_S}}_{>0} > 0$

(more clouds means a more opaque atmosphere and so a higher emission level)

(iii) defining $f_{\alpha_S} = -\frac{S_0(1+c)}{16\sigma T_E^3} \underbrace{\frac{d\alpha_S}{dT_S}}_{<0} > 0$

(increasing T_S means less sea-ice so more solar radiation absorbed)

<u>NB</u>: this is the surface albedo feedback discussed in lecture 13

we rewrite dT_s equation as

$$dT_s = f_{\alpha_s}\, dT_s + (f_{C_{neg}} + f_{C_{pos}})\, dT_s + \Gamma\left(\frac{\partial z_E}{\partial CO_2} dCO_2 + \frac{\partial z_E}{\partial H_2O} dH_2O\right)$$

we recognize the water vapour feedback

$$f_{H_2O} \equiv \Gamma \frac{\partial z_E}{\partial H_2O} \frac{\partial H_2O}{\partial T_s}$$

$$dT_s = (f_{\alpha_s} + \underbrace{f_{C_{neg}} + f_{C_{pos}}}_{\equiv f_C} + f_{H_2O})\, dT_s + \Gamma \frac{\partial z_E}{\partial CO_2} dCO_2$$

integrating the latter from $1\times CO_2$ world to $2\times CO_2$ world:

$$\Delta T_s = \frac{\Delta T_s^{(ref)}}{1 - f_{\alpha_s} - f_{H_2O} - f_C} \quad \text{where } \Delta T_s^{(ref)} = \underbrace{\Gamma \frac{\partial z_E}{\partial CO_2}}_{\sim 1K} \underbrace{\Delta CO_2}_{2\times CO_2}$$

③ $$\Delta T_s = \frac{\Delta T_s^{(ref)}}{1 - f_{H_2O} - f_{other}}$$

for $\begin{cases} \Delta T_s^{(ref)} = 1K \\ f_{H_2O} = 0.4 \end{cases}$ $\quad 1.5K \le \Delta T_s \le 4.5K$

$$1.5K \le \frac{1K}{1 - 0.4 - f_{other}} \le 4.5K$$

this means that some models have a strong net positive feedback (other than f_{H_2O}) in order to predict $\Delta T_s = 4.5K$

$$\frac{1}{0.6 - f_{other}} = 4.5 \Rightarrow f_{other} = +0.37 = f_{other}^{+}$$

Conversely, to predict $\Delta T_s = 1.5\,K$ some models must have a weak net negative feedback :

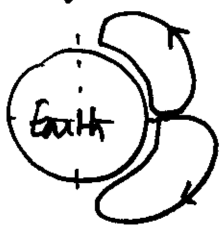
$$\frac{1}{0.6 - f_{other}} = 1.5 \Rightarrow f_{other} = -0.06 = f^{-}_{other}$$

$\therefore$ if there was no water vapour feedback ($f_{H_2O} = 0$) the spread in models would be reduced :

$$\frac{\Delta T_s^{(ref)}}{1 - f^{-}_{other}} \leq \Delta T_s \leq \frac{\Delta T_s^{(ref)}}{1 - f^{+}_{other}} \quad \text{i.e} \quad \underline{0.94\,K \leq \Delta T_s \leq 1.59\,K}$$

Water vapour feedback not only increases ΔT_s for a $2\times CO_2$, it also makes the climate more sensitive to other feedbacks.

④ (i)



rate at which mechanical energy is produced by one cell is

$$4 \times W \quad [J s^{-1}]$$

The surface area it occupies is

$$2\pi \times R^2$$

$\therefore$ $\frac{4W}{2\pi R^2}$ is the rate at which mechanical energy is produced per unit area $[W m^{-2}]$

(ii) $\frac{4Q}{2\pi R^2}$ is similarly the rate of heating per unit area $[W m^{-2}]$. If we assume that all of it comes from evaporating sea water then $\frac{4Q}{2\pi R^2} = F \equiv$ enthalpy flux at Earth surface.

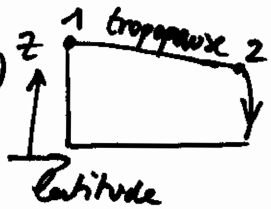
(iii) $W = \eta \dot{Q} \rightarrow \frac{W}{2\pi R^2} = \eta \frac{\dot{Q}}{2\pi R^2} = \eta F$

$\nearrow = \dot{W} \qquad \therefore \eta = \dot{W}/F$

(iv) In steady state, production of mechanical energy = dissipation of mechanical energy = $1\,\mathrm{Wm^{-2}}$

$\rightarrow \eta \sim \frac{1\,\mathrm{Wm^{-2}}}{100\,\mathrm{Wm^{-2}}} \sim 1\% \ll \frac{T_s - T_t}{T_s} = \frac{288-210}{288} = 25\%$

⑤ (a)



if only pressure changes from 1 to 2 are considered:

$\Delta S_{(p)} = -R \ln \frac{p_2}{p_1} = -R \ln \frac{300}{100} = -315\,\mathrm{JK^{-1}kg^{-1}}$

if only temperature changes are considered:

$\Delta S_{(T)} = c_p \ln T_2/T_1 \sim c_p \ln \frac{215}{205} \sim +47\,\mathrm{JK^{-1}kg^{-1}}$

$\therefore \Delta S_{total} = \Delta S_{(p)} + \Delta S_{(T)} = -268\,\mathrm{JK^{-1}kg^{-1}} < 0$

Pressure effects dominate: when going from 1 to 2 the parcel pressure increases so its entropy decreases. The error committed in neglecting $\Delta S_{(T)}$ is on the order of $\frac{\Delta S_{(T)}}{\Delta S_{(p)} + \Delta S_{(T)}} \sim 17\%$

(b)

$\dot{Q} = -T_s \Delta S_{total} \simeq +10^5\,\mathrm{J\,kg^{-1}}$

(c) In a closed loop, the change in Potential energy (PE) is zero since the initial and final positions of the parcel are the same ($\oint d\Phi = 0$)