Environmental Physics

Problem Sheet 5: Photovoltaic energy conversion

1. An ideal photovoltaic converter of band gap E_g in full concentration generates electrons in an external circuit with a flux density of $\frac{1}{e}J = \int_{E_g}^{\infty} b(E,T_{sun})dE - e^{\Delta\mu/kT}\int_{E_g}^{\infty} b(E,T_p)dE$ where $b(E, T)$ represents the spectral photon flux density from a black body at temperature T.
(a) Explain qualitatively what will happen to (i) the photocurrent density at short circuit, J_{SC} (ii) the voltage V_{OC} at which current = 0 and (iii) the maximum efficiency, as the band gap E_g is varied. You may assume that $\Delta\mu = eV$.
(b) Now suppose that the ideal converter can only absorb or emit light with energies in the range E_g to $E_g + \Delta E$. Discuss what will happen to the maximum efficiency in this case.

2. In a semiconductor at equilibrium, the electron density n_0 and hole density p_0 are given by $n_0 = n_C e^{-(E_C-\mu)/kT}$ and $p_0 = n_V e^{-(\mu-E_V)/kT}$.
(a) Show that $n_0 p_0 = n_i^2$ where $n_i = \left(n_C n_V e^{-(E_C-E_V)/kT}\right)^{1/2}$

(b) In a p-doped semiconductor $p_0 = N_A$ ($>> n_i$) when the acceptor impurities are easily ionised at room temperature. Show that the equilibrium electron density is given by $n_0 = n_i^2 / N_A$. Find n_0 and p_0 for silicon ($n_i = 1\times10^{16}$ m^{-3} at 300 K) when doped with a density of 1×10^{23} boron atoms.

(c) Light of wavelength 800 nm and power density 0.1 W m^{-2} is incident on a slab of p-type silicon 100 μm thick. If the linear absorption coefficient α for silicon is 7×10^4 m^{-1} at 800 nm and reflectivity is negligible, find the charge generation rate in the slab [in units of m^{-3} s^{-1}]. If electrons have a lifetime of 10^{-6} s in this material, find the steady state electron density under this irradiation. If the light intensity is increased to 10 W m^{-2}, find the amount by which the chemical potential of the electrons, μ_n, increases. You may assume that under illumination, the electron density obeys $n \propto e^{\mu_n/kT}$.

3. Verify that the expression $n(x) = Gt_n\left\{1 - \frac{\cosh((x-d)/L)}{\cosh(d/L)}\right\}$ is a solution to the continuity equation for electrons $D_n\frac{d^2n}{dx} - \frac{n}{t_n} = -G$ with the boundary conditions $n(0) = 0$ and $J_n(d) = 0$, where $L = \sqrt{D_n t_n}$.
Explain what happens to the electron density at the blocking contact as (i) the optical depth of the sample is varied (ii) the electron diffusion length L is varied.

4. Monochromatic light of energy E and photon flux density F_0 is incident on a slab of p-type semiconductor of absorption coefficient α and reflectivity r. Write down an expression for the photon flux density $F(x)$ of photons of energy E as function of depth x into the slab. Hence find the photon flux density that will be absorbed by the slab if it has thickness d. Assuming that the slab has perfectly asymmetric contacts to the outside world, what is the maximum photocurrent that the slab can generate if the electron diffusion length is (i) $L >> d$? (ii) $L << d$?
Find expressions for the photocurrent quantum efficiency (QE) in each case.
(QE = flux density of electrons collected / flux density of incident photons)

5. (a) Show that for an ideal diode solar cell, $V_{oc}(X) = kT\ln(X) + V_{oc}(1)$ where $V_{oc}(1)$ represents the open circuit voltage in standard solar radiation ($X = 1$).

(b) A photovoltaic device is optimised for a load of resistance R_L in standard solar radiation. Use the diode equation fo a solar cell to explain what happens to the current, voltage and efficiency when the light intensity is reduced if R_L is kept the same. If intensity is reduced by a factor of 10, estimate the factor by which efficiency is reduced (i) if R_L is not changed (ii) if R_L is optimised for the new intensity.

6. (a) Balance the following equation for the production of sucrose:

$$x\mathrm{CO_2} + y\mathrm{H_2O} + \mathrm{h}\nu \rightarrow \mathrm{C_{12}H_{22}O_{11}} + z\mathrm{O_2} + w\mathrm{H_2O}$$

(b) Find the combustible energy density in sucrose if the enthalpy per C atom is 4.8 eV.
[You may take the atomic masses of H, C and O to be 1, 12 and 16 respectively; a mole contains 6.02×10^{23} molecules and a mole of H weighs 1g.]

7. A land area of 10^5 m^2 with an average solar irradiance of 150 W m^{-2} is to be used to support a population of humans. The average energy requirement per capita is 150 W food energy, 200 W heat and 250 W electricity. It is proposed to provide the food energy via a food crop, the heat by combustion of biomass (dry wood) and the electricity via photovoltaic panels. (a) Use the data given below to calculate the maximum population that can be supported sustainably by the land area in this way.
(b) As an alternative, it is proposed to increase the population density by providing the entire heat requirement by photovoltaic electricity. How many people could now be supported? Identify two reasons why this number is unrealistic.

Data:

Annual average photosynthetic efficiency for food crop	2%
Dry wood yield from temperate forest	7,000 kg ha^{-1} year^{-1}
Energy density in dry wood	18 MJ kg^{-1}
Power conversion efficiency of photovoltaic panels	10%