Environmental Physics. Problem Sheet 4: Conversion of solar energy

1. The spectral power density emitted by a black body at temperature T (Planck's radiation law) is often given in terms of wavelength *i.e.* $u(\lambda)d\lambda = \frac{2\pi hc^2}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$.
Show by equating elements of the distributions $u(\lambda)d\lambda = u(E)dE$ that this is equivalent to the form of the law in terms of energy, $u(E)$, that we used in lectures.

2. Use the information below to calculate the solar constant and the expected emission temperature T_p of the planets Mars, Earth, Venus, Jupiter, neglecting the effects of atmosphere in each case. Comment on the predicted temperature compared with the actual temperature.

Planet	Distance from sun / m	Actual temperature / K
Venus	1.08×10^{11}	733
Earth	1.50×10^{11}	288
Mars	2.27×10^{11}	220
Jupiter	7.78×10^{11}	124
Radius of Sun: 6.96×10^{8} m		

3. (a) The mean energy consumption per capita is around 5 kW in Western Europe. Calculate the land area required to supply the energy needs of a city of 10 million people using solar irradiation in an area with mean solar irradiance of 125 W m^{-2}, if solar energy can be converted into useable energy with a net efficiency of 10%.

(b) What land area would be needed to supply the needs of a community of 100,000 people using energy at the global average rate of 2 kW, in an area with mean solar irradiance of 250 W m^{-2}?

4. Show that the angle subtended by the Sun to the Earth is $\theta_{sun} = 0.26°$. Show that the maximum concentration ratio that can be achieved with one-dimensional concentration is $X = 1/\sin(\theta_{sun})$. What practical factors could limit the concentration ratio available for a parabolic trough focussing light on to a tube of fluid?

5. The efficiency of an ideal solar thermal converter receiving solar radiation from the sun at temperature T_{sun} at concentration X is given by:

$$\eta = \left\{1 + \frac{1 - Xf_s}{Xf_s}\left(\frac{T_p}{T_{sun}}\right)^4 - \frac{1}{Xf_s}\left(\frac{T_c}{T_{sun}}\right)^4\right\}\left(1 - \frac{T_p}{T_c}\right) \qquad T_{sun} > T_c > T_p$$

Find expressions for the lower and upper limits of the range of temperatures T_c for which the converter generates power. Use Excel (or a similar program) to calculate η for an ideal solar thermal converter as a function of the converter temperature T_c and plot the results for X =1, X = 200, X = 4000. What is the maximum power conversion efficiency in each case and at which T_c does it occur? In a practical solar thermal electric converter, how can T_c be controlled? (You may assume the Sun temperature T_{sun} = 5780 K and the ambient temperature T_p = 300 K, and $f_s = 2.16\times10^{-5}$).

6. A Brayton gas cycle uses the following four stages: 1→2: adiabatic compression. 2→3: isobaric heat supply. 3→4: adiabatic expansion. 4→1: isobaric heat rejection. Sketch the P-V and T-S diagrams for the cycle. By considering the change in internal energy for the heating or cooling of a gas at constant pressure, show that the efficiency of the engine is given by $\eta = 1 - (T_4 - T_1)/(T_3 - T_2)$. Can this ever be larger than the efficiency of a Carnot engine working between the temperatures T_3 and T_1? Hint: Consider the area enclosed by the T-S diagram.

7. If the heat engine in Q.5 were an irreversible engine (such as a Brayton engine) rather than a Carnot engine, what would you expect to happen to the optimum values of T_c and η at a given concentration X? Explain your answer.