Clausius-Clapeyron scaling

From Thermodynamic 2nd year, the pressure of water vapour at equilibrium (denoted here by the symbol e_s for 'saturation vapour pressure') with liquid water satisfies, at a given temperature T,

$$\frac{de_s}{dT} = \frac{s_v - s_l}{v_v - v_l} \quad \text{(equation for the gas - liquid phase boundary)} \tag{1}$$

in which s is the specific entropy, v the specific volume and the subscripts l and v refer to liquid and gas phases, respectively. Since by definition $s_v - s_l = l_v/T$ in which l_v is the latent heat of vaporization and given that $v_v \gg v_l$,

$$\frac{de_s}{dT} \simeq \frac{l_v}{Tv_v} \tag{2}$$

Treating water vapour as an ideal gas, $e_s v_v = N k_B T / (N m_v) = k_B T / m_v$ in which N is the number of water vapour molecules, each of mass m_v and k_B is the Boltzmann constant, this can be rewritten as

$$\frac{de_s}{dT} \simeq \frac{e_s l_v}{R_v T^2} \tag{3}$$

in which we have further introduced the gas constant for water vapour $R_v = k_B/m_v$. Rearranging, we finally obtain

$$\frac{de_s}{e_s} \simeq \frac{l_v}{R_v T^2} dT \tag{4}$$

This is the basis for the 7%/K increase in atmospheric water vapour content discussed in Lecture 8.