Collisionless shocks

Lecture 1 (10:00 – 10:50)
Lecture 2 (11:10 – 12:00)
Lecture 3 (13:45 – 14:45)

• Introduction to shocks
• Different ways of looking at a plasma
• MHD shocks – Rankine-Hugoniot relations

• Real shocks
  • Shock orientation
  • Importance of B
  • Foreshock
  • Particles
  • Quasi-parallel shock

Friday 14:00 – 15:00 Discussion of problems
Why do shocks form?

- They solve an information flow problem
- A disturbance in a fluid emits waves
  - Wave speed $c_s$
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- An obstacle in subsonic flow
  - Waves communicate presence of obstacle to fluid
  - Fluid flows around obstacle
Why do shocks form?

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• An obstacle in supersonic flow
  • $V_{flow} > c_s$
  • Sound waves swept back by fluid
  • Shock formed
    • $V_{shock} > V_{flow}$
    • Slows and deflects flow
    • $E_{flow} \rightarrow$ dissipated as heat
Shape of shock wave

- Shape of shock depends on shape of obstacle
- Blunt obstacles tend to have shock standing away from surface
- Flaring of shock surface greater for more blunt object
How do shocks form?

- Consider a sound wave in a gas
  - Compressional wave – perturbation in pressure $\Delta P$
    
    \[
    c_s = \sqrt{\frac{\gamma P}{\rho}} \quad \text{where} \quad \gamma = \frac{C_p}{C_v}
    \]

- Small $\Delta P$
  - No significant change in $c_s$ within wave
  - Wave profile does not evolve

- Large $\Delta P$
  - Wave steepens
  - Shock forms
  - Mach number
    
    \[
    M = \frac{V_{\text{flow}}}{c_s}
    \]
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  • Mach number = shock strength

$$M = \frac{V_{flow}}{c_s}$$

• Analogous to waves breaking on a beach
Collisionless shocks

• Ubiquitous throughout universe
  – Bow shocks upstream of stars
Collisionless shocks

- Ubiquitous throughout universe
  - Bow shocks upstream of stars
  - Shocks generated by supernova

SNR 1572 “Tycho”
Collisionless shocks

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  - Corotating interaction regions
    - Forward shock
      - Speeds up slow solar wind
    - Reverse shock
      - Slows down fast solar wind

- Why are they interesting?
  - Change large scale patterns of flow
  - Heat plasma
  - Accelerate particles
What happens at a shock?

• Consider ideal “thin” shock
• Propagating/standing shock?
  • Work in rest frame of shock
• Mass is conserved
• Momentum is conserved
• Energy is conserved
• Internal distribution of energy given by
  • Equation of state
  • Easy for Ideal gases \( PV = nkT \quad \Rightarrow \quad P = \rho kT \)
  • Hard for collisionless plasmas
    • Multiple wave modes
    • Multiple species
    • No collisions – far from equilibrium
    • Multiple options for energy partition – no unique solution
What happens to the plasma as it crosses the shock?

• Model dependent…

• Exact particle density function: $F(t, x, v)$ with microscale $E_m$ and $B_m$ fields
  – Easy to derive – hard (impossible) to solve
  – Treats motion of every particle individually
  – *Simplify* – *don’t expect the motion of every particle to be independent of those nearby*
  – Klimontovich-Dupree equation

• Kinetic description: $F(t, x)$ obtained by averaging over ensemble of particles
  – Assume:
    • $F = \langle F \rangle \pm \delta F$ where $\langle \delta F \rangle = 0$
    • $E = \langle E \rangle \pm \delta E$ and $B = \langle B \rangle \pm \delta B$ where $\langle \delta E \rangle = 0$ and $\langle \delta B \rangle = 0$
  – Average over ensemble to get $F(t, x)$
  – Hard because $E_m$ and $B_m$ are not independent of $F$
Further approximations to the kinetic approach

- **Boltzmann equation**: separate acceleration term into a term averaged over the ensemble plus a term which represents all particle correlations and interactions
  - Form of collision term depends on parameter regime and assumptions

- **Fokker-Planck equation**: assume NO collisions, but then particle correlations are important
  - Inter-particle interactions represented by a diffusion term

- **Vlasov equation**: Assume no interactions of any kind
  - Collision term = 0
  
  \[
  \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_x f + \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_v f = 0
  \]
  - Phase space volume can be deformed but not changed
  - \( f(\mathbf{x}, \mathbf{v}, t) \) is constant along an trajectory in phase space
  - Liouville’s theorem – true for electrons across the bow shock (but not ions)
Moments of the particle distribution function

• Relate particle distribution function to measurable quantities by taking moments:
  – Multiply by $v$ and integrate over velocity space

• Zeroth: number density
  \[ n = \int f(v) d^3v \]

• First: bulk velocity
  \[ v_b = \int vf(v) d^3v \]

• Second: pressure $P$, contributions from velocity fluctuations of ensemble from the mean
  \[ P = m \int (v - v_b)(v - v_b) f(v) d^3v \]
  – In general $P$ is a tensor; if pressure is isotropic then $P$ is related to scalar $p$ by:
    \[ P = \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} \quad p = \frac{1}{3} \text{Trace } P \quad \text{Where, in an ideal gas } \quad p = \frac{1}{3} mnv^2 = \rho kT \]

• Third: heat flux, but normally close equations using equation of state relating $p$ and $V$
  – Closing equations introduces assumptions – e.g. ideal gas (but measured $\gamma$ seen to vary across a shock)
Back to shocks… what happens to the plasma?

• Take a single fluid approximation to the plasma
  – Mass density $\rho$
  – Flow velocity $\mathbf{v}$
  – Pressure $p$ (isotropic)

• Magneto-hydrodynamic equations (MHD) represent conservation of mass, momentum and energy in a flowing plasma
  – Assume static, 1D shock
  – Assume plasma acts as a perfect gas
  – Assume plasma reacts adiabatically
  – Can use energy equation
    OR
  – Equation of state

\[
\frac{p}{\rho^\gamma} = \text{constant}
\]

\[
\text{i.e. } \frac{d}{dn} \left( \frac{p}{\rho^\gamma} \right) = 0 \quad \text{or} \quad \left( \frac{p_u}{\rho_u^\gamma} \right) - \left( \frac{p_d}{\rho_d^\gamma} \right) = 0 \quad \text{or} \quad \left[ \frac{p}{\rho^\gamma} \right] = 0
\]
Rankine-Hugoniot relations

Mass conservation

Continuity of $B_n$

Continuity of $E_t$

Energy conservation

Momentum conservation $\parallel n$

Momentum conservation $\perp n$

- R-H relations true for all discontinuities, not just shocks
- If inflow velocity $\rightarrow$ subsonic; solutions revert to MHD waves: fast, slow, Alfvén
Fast mode perpendicular and parallel shocks

• Exactly perpendicular shock
  – Solve R-H to show compression ratio
    \[ r = \frac{\rho_d}{\rho_u} \]
  – Strong shock limit (high Mach number)
    \[ r = \frac{\gamma + 1}{\gamma - 1} \]
    Where if \( \gamma = 5/3 \); \( r_{\text{max}} = 4 \)
    – (See exercise 1)

• Exactly parallel shock
  – B drops out of R-H
  – Same as a gas dynamic shock
  – Real parallel shocks don’t look anything like this!
Fast shocks

- Most shocks are fast mode
  \[ M_{MS} = \frac{u_u}{c_{MS}} = \frac{u_u}{\sqrt{c_s^2 + v_A^2}} \]
  \[ v_A^2 = \frac{B^2}{\mu_0 \rho_u} \]
  \[ c_s^2 = \frac{\gamma \rho_u}{\rho_u} \]

- \( M_{MS} \) often hard to calculate so often use the Alfvén Mach number
  \[ M_A = \frac{u_u}{v_A} \]

- Density rises
- Magnetic field increases
  - \( B_n = \) constant
  - \( \mathbf{B} \) turns away from shock normal \( \mathbf{n} \)
Slow shocks

- Slow shocks
  - Rare – associated with reconnection
  - Density rises
  - Magnetic field decreases
    * $B_n = \text{constant}$
    * $\mathbf{B}$ turns towards shock normal $\mathbf{n}$

- Sound speed: $c_s^2 = \frac{\mathcal{P}_u}{\rho_u}$

- Sonic Mach number: $M_{cs} = \frac{u_u}{c_s}$
Practical application to real shocks: shock normal and velocity

• Coplanarity
  – For oblique shocks can show that $B_u$, $B_d$ and $n$ are coplanar
    • Observationally true in absence of $E_n$ (away from transition)
  – Since divergence of $B$ is zero

  – Used to calculate shock normal
  – Can also be used to calculate normal to tangential discontinuity

\[
\left( B_u - B_d \right) \cdot \hat{n} = 0
\]

\[
\left( B_u \times B_d \right) \cdot \hat{n} = 0
\]

\[
\hat{n} = \frac{\left( B_u - B_d \right) \times \left( B_u \times B_d \right)}{\left| B_u - B_d \right| \left( B_u \times B_d \right)}
\]

• Mixed mode normal
  – Need $B$ and $v$ measurements
  – $\Delta v =$ change in velocity from u/s to d/s

\[
\left( B_u \times \Delta v \right) \cdot n = 0
\]

\[
\left( B_d \times \Delta v \right) \cdot n = 0
\]

$\Delta B \cdot n = 0$
Practical application to real shocks: shock normal and velocity

- Four-spacecraft normal and velocity
  - Assume shock is planar, moving with constant $v$, then
  - If have
    $$\delta x_i (i = 1, 2, 3, 4)$$
  - Then can calculate
    $$\delta x_i \cdot n = \dot{v} \cdot n \delta t_i$$
  - NB Only shock velocity parallel to $n$

- Shock velocity
  - Shock normal, determined from either coplanarity or mixed mode
    +
  - Conservation of mass across shock

\[
v_{sh} = \frac{\rho_d u_d - \rho_u u_u}{\rho_d - \rho_u} \cdot \hat{n}
\]
Real shocks – must take account of particles

**Perpendicular**

\( \theta_{Bn} \sim 90^\circ \)

**Parallel**

\( \theta_{Bn} \sim 0^\circ \)

\[ B \]

Flow

Shock

Ions

\( n \)

\( \approx 90^\circ \)

\( \approx 0^\circ \)

Flow

Shock

\[ N_p \] (cm^-3)

\[ V \] (km/s)

|B| (nT)

2001 Mar 30, hour min (UT)

19 15 19 30 19 45 20 00 20 15

2002 Feb 20, hour min (UT)

15 00 15 30 16 00 16 30 17 00 17 30 18 00
Quasi-perpendicular shocks ($\theta_{Bn}$~90°)

Schematic...

- $\Delta B$ supported by $j_{sh}$
- $E_{sh}$ from charge separation
  - Reflects some ions (fraction depends on Mach no)
  - Attracts/captures $e^-$
- Reflected ions
  - Gyrate
  - Accelerated by motional $E_{sw}$
  - Form shock foot
  - Carry current $j_f$
  - Re-cross shock
  - Gyrate downstream
  - Generate waves
  - Thermalise plasma
- Electrons drift in $d_{cs}$
  - Contribute to $|B|$ overshoot

Figure after Baumjohann and Treumann. 1997 (not to scale)
Different types of $Q_\perp$ shock: the importance of Mach number

- Low Mach number: 1 - 3 e.g. Interplanetary shocks:
  - Sharp transition; little overshoot
  - Sometimes see clear dispersive whistler leading shock ramp

- Moderate to High Mach number: 3 - 10 e.g. Terrestrial bow shock:
  - Foot; Ramp; Overshoot; Downstream wave decay
  - Ion reflection plays essential role

- High Mach number e.g. some astrophysical shocks
  - Other mechanisms become important
The Earth’s bow shock

- Curved geometry
  - Shock orientation ($\theta_{Bn}$) varies

- Electrons and ions reflected/escape to form foreshock
  - Morphology depends on $B$, particle energy, cross-field drift speed
  - Not steady state but time varying

- Upstream particles generate waves
  - ULF wave foreshock

- ULF waves convect back towards the shock
  - Potential to modify shock geometry
  - Seen in modulation of reflected ion populations

- Rankine-Hugoniot only true if include foreshock

From Treumann and Scholer, 2001
Research topics

• Quasi-perpendicular shocks
  – Shock orientation – real measurements
  – Shock variability
  – Substructure within the ramp
  – Source of ions beams upstream

• Quasi-parallel shocks
  – Ion acceleration
  – How the parallel shock works