Answers to Problems for Chapter 1

Q1

(i) Winter pole, $25km$ for the stratosphere but Summer pole, $80km$ for the mesosphere. The lower stratosphere is also very cold year round at the equator, and this reflects the deep penetration of convective motions and the tendency to follow a moist adiabatic structure at low latitudes (this will be discussed in Chapter 3).

(ii) Summer pole near the stratopause for the stratosphere, but Winter pole for the mesosphere (particularly warm right at the stratopause but hot throughout the depth of the mesosphere). This surprising feature, and the cold Summer pole in (i), reflect a planetary circulation extending from the Summer to the Winter poles. In the ascending branch (Summer pole), air cools by doing work of expansion against its surroundings, hence the low temperature there. Likewise, in the descending branch, air is compressed and warms up, hence the high temperature there.

(iii) For both stratosphere and mesosphere, the strongest westerly winds (i.e., from west to east) are found in the Winter Hemisphere (the "polar vortex" for the stratosphere, and much closer to the equator for the mesosphere –near 30° of latitude).

Q2 This is because of the descending motion associated with the Hadley cell in the sub-tropics. This brings dry air from high levels to the surface which prevents any cloud and rain formation.

Q3

(i) (a) The mass mixing ratio of ozone (r_{O_3}) is the ratio of the mass of ozone in a sample of air to the mass of dry air in the sample, $r_{O_3} = m_{O_3}/m_d$. The density of ozone is $\rho_{O_3} = m_{O_3}/V$ where V is the volume of the sample. We can, as far as pressures are concerned, neglect the presence of water vapour and ozone in the stratosphere and treat the sample as "dry air", i.e., $P_o = P_d + P_{O_3} + e \approx P_d$. Hence $P_o = \rho_d R_d T_o$ where ρ_d is the density of dry air. From $\rho_{O_3} = r_{O_3}\rho_d$, we get $\rho_{O_3} = r_{O_3}P_o/R_dT_o \approx 4 \times 10^{-7} kgm^{-3}$. (b) Using the ideal gas law, $P_{O_3} = \rho_{O_3} R_{O_3} T_o$ in which $R_{O_3} = k_B/\mu_{O_3}$ is the gas constant for ozone. To obtain the latter, use $R_d = k_B/\mu_d$ so that $R_{O_3} = R_d\mu_d/\mu_{O_3} \approx 287 \times \frac{(0.8\times28+0.2\times32)}{48} \approx 172.2 JK^{-1}kg^{-1}$. This yields $P_{O_3} \approx 0.015 Pa$.

(ii) At a given temperature T and volume V, one has $e/P_d = N_v/N_d$ using the ideal gas law and $N_v/N_d = 10/1000 = 0.01$. Hence water vapour molecules account for $N_v/(N_d+N_v) = 0.01/(1+0.01) = 0.0099\% \approx 1\%$ of air molecules in the sample. Rewriting $1\% = 10^{-2} = 10^4 \times 10^{-6} = 10^4\ ppm$, we see that this fraction is $10^4/400 = 25$ times that of CO_2.

Q4

(i) The new airmass weights $2kg$ and contains $5+10 = 15g$ of water vapour. Hence $q_3 = 15/2 = 7.5kgm^{-3}$. More generally, if $x = m_1/m_2$ (the ratio of the two masses of gas), one has $q_3 = q_1/(1+1/x) + q_2/(1+x) = (xq_1 + q_2)/(1+x)$.

(ii) Specific humidity is an intensive variable since in the case in which $q_1 = q_2$, one obtains after mixing $q_3 = q_1 = q_2$ (independently of x) rather than $q_3 = q_1 + q_2$. In other words it is not proportional to the mass of the sample.

Q5

(i) Using the hydrostatic equation $\partial P/\partial z = -\rho g$ in which ρ is density. For an isothermal atmosphere, the ideal gas law reads $P = \rho(k_B/m)T_o$ so that $\partial P/\partial z = -Pmg/k_BT_o$. This proves $p \propto e^{-z/H_s}$ with $H_s = k_BT_o/mg$ as required.

(ii) One needs to estimate first the value of gravity at the planet's surface, which can be done using Newton's aw of gravitation, $g \approx GM/R^2$ in which M is the planet's mass and R its radius. Then it is just a matter of plugging in the numbers. One gets: $g = 8.87(V), 9.81(E), 3.71(M), 24.8(J)ms^{-2}$ and the resulting scale heights are, $16(V), 8.2(E), 12.7(M), 20.6(J)km$.

Q6 Start from $\alpha = (V_g + V_l + V_i)/(m_d + m_v + m_l + m_i)$ (in which V_g is the volume occupied by the gas phase, V_l, V_i by the liquid and ice phases, respectively) factorize by m_d to obtain $\alpha = (V_g/m_d + V_l/m_d + V_i/m_d)/(1+r_t)$ in which $r_t = (m_v + m_l + m_i)/m_d$ is the total mass mixing ratio of water substance. Acknowledging that $\alpha_d = V_g/m_d$, this can be rewritten as, $\alpha = (\alpha_d + V_l/m_d + V_i/m_d)/(1+r_t)$. Introducing $\alpha_l = V_l/m_l$ and $\alpha_i = V_i/m_i$, one gets, $\alpha = \alpha_d(1 + r_l\alpha_l/\alpha_d + r_i\alpha_i/\alpha_d)/(1+r_t)$. Both r_l and r_i are very small compared to unity, as are α_l/α_d and α_i/α_d, so $\alpha_d \approx \alpha_d/(1+r_t)$. Since $1 + r_t = 1/(1-q_t)$, the result follows.

Q7 Higher pressure is found on the western than on the eastern side of the Rockies. The atmosphere is thus "pushing" the Rockies towards the east. From Newton's 3rd law, the Rockies must thus "push" the atmosphere towards the west. The Rockies thus contribute to a loss of angular momentum (same sign as the loss due to surface friction). NB: The pressure pattern is somewhat easier to see for the Andes on the ERA website. This probably reflects the Taylor column effect partly at work in the Rockies (the wind going around the mountain), but less so over the Andes because they occupy a broader range of latitudes (air parcels have "nowhere to go" but over the mountain).

Q8 Assuming $u \approx 0$ at the equator, the angular momentum of the ring as it is about to go upward is $L = \Omega R^2$. Assuming L is conserved all the way during the ascent and the poleward motion, we have, $\Omega R^2 = R\cos\phi(\Omega R\cos\phi + u)$. This provides a formula for u as a function of latitude ϕ, namely, $u = \Omega R\sin^2\phi/\cos\phi$. At $30°N$ this provides, $u \approx 134ms^{-1}$! Such velocities are not observed because the ring breaks up in waves (the storms) before reaching this value

Q9 The incoming solar radiation is estimated at $340.2Wm^{-2}$, cloud absorption in the short wave at $5Wm^{-2}$, and atmospheric absorption in the short wave at $75Wm^{-2}$. Thus the fraction absorbed by the atmosphere and cloud is $(75+5)/340.2 = 0.24$. Another way to estimate this heating is to look at the convergence between the net downward at the top $(340.2 - 100 = 240.2)$ and that at the surface $(165 - 23 = 142)$, leading to an absorption of $240.2 - 142 = 98Wm^{-2}$ and a fraction $98/340.2 = 0.29$. This is $18Wm^{-2}$ larger than the $75 + 5 = 80Wm^{-2}$ quoted in the text (I suspect that this is because they have chosen to close the total heat budget rather than each separate vertical component). In any case, the fraction absorbed is not entirely negligible but small enough for a zero order view.