Solution to Chapter 5's problems

Q1. First compute the energy associated with either the presence of sea ice and a $0.1K$ increase in global ocean temperature. For the sea ice, this energy (in Jm^{-2}) is $\rho_i h_i l_f$ where ρ_i is ice density, h_i is ice thickness and l_f is the latent heat of fusion for water. For the ocean temperature change, this energy is $\rho_o c_o h_o \Delta T$ in which ρ_o is ocean density, c_o is the specific heat capacity of the oceans, $\Delta T = 0.1K$ and h_o is the averaged depth of the ocean.

Putting numbers, one finds, $\rho_i h_i l_f = 0.5 \times 10^9 Jm^{-2}$ and $\rho_o c_o h_o \Delta T = 1.5 \times 10^9 Jm^{-2}$. The time it would take to melt the sea ice would thus be $0.5 \times 10^9 Jm^{-2}/0.5Wm^{-2} = 35\ years$ and it would take about three times longer ($\simeq 100\ years$) to increase the global ocean temperature by $0.1K$. The point is that complete disappearance of the sea ice represents a small amount of energy in comparison to ocean temperature rise: it is equivalent to a global ocean temperature increase of $0.03K$ only.

NB: Obviously the scenario used here is somewhat unrealistic for the ocean since the infrared flux would only be felt at the surface not through the whole depth of the ocean –the warming at depth would penetrate due to transport by currents and this could take longer than 100 $years$. Note also the absence of feedbacks: were the sea ice to disappear, the TOA radiative fluxes would change since the planetary albedo would decrease accordingly.

Q2.

(i) The emission level is simply, according to the definition given in the question, the height at which temperature equals the emission temperature. From Chapter 2, this temperature is $T_E = 255K$. If the surface temperature is $288K$ and the lapse-rate is $7K/km$, this height corresponds to $(288 - 255)/7 = 4.7km$.

(ii) When CO_2 is increased, so does the opacity of the atmosphere in the infrared. The OLR is accordingly weakened and the radiation emitted by the Earth must come from "higher up" in the troposphere where the temperature is smaller. Hence an increase in the emission level (Fig. 1).

(iii) At equilibrium, the OLR must still be the same than before the CO_2 perturbation is applied, assuming the absorbed solar flux does not change. Thus the temperature at the new height ($z_e + \delta z_e$) of the emission level must still be $255K$. Fig. 1 illustrates the old (T_o, solid curve) and new (T_1, dashed curve) temperature profiles. Assum-

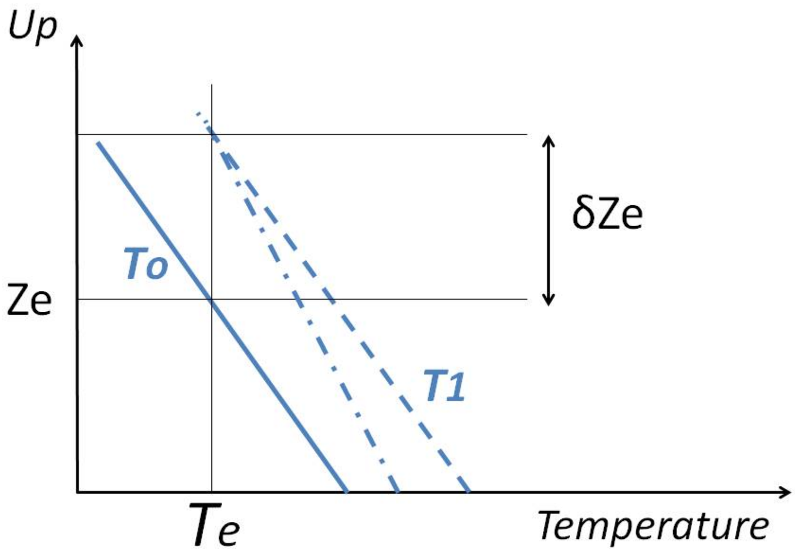

Figure 1: Schematic of temperature as a function of height, indicating the emission temperature and emission level. The increase in emission level in response to a sudden doubling of CO_2 is also indicated, as well as the correpsonding change in temperature (dot-dashed and dashed curves).

ing the lapse rate is unchanged and a $3K$ warming, one must have: $3K/\delta z_e = 7K/km$. This yields an increase $\delta z_e \approx 420m$.

(iv) Fig. 1 illustrates the solution. Start at $z_e + \delta z_e$, where the temperature is the same as in the "no lapse-rate change" case, i.e., equal to T_e. Since the lapse-rate decreases with surface warming, the new temperature profile must decrease less sharply with height than T_1 does, as indicated on the diagram (dot-dashed line). As can be seen, the surface temperature change is accordingly smaller. The lapse-rate feedback is negative.

Q3.

(i) By definition $s_v - s_l = l_v/T$ in which l_v is the latent heat of vaporization and $v_v \gg v_l$; the result follows.

(ii) Treating water vapour as an ideal gas, $e_{eq}v_v = Nk_BT/(Nm_v) = k_BT/m_v$ in which N is the number of water vapour molecules, each of mass m_v and k_B is the Boltzmann constant. The result follows.

(iii) Rearranging, we obtain

$$\frac{de_{eq}}{e_{eq}} \simeq \frac{l_v}{R_vT^2}dT \tag{1}$$

For $l_v = 2.5\ 10^6 Jkg^{-1}$, $R_v = 461 Jkg^{-1}K^{-1}$ and $T = T_s = 288K$, we find $de_{eq}/e_{eq} \simeq 0.07K^{-1}$, or, an increase in equilibrium vapour pressure of 7% per degree K warming.

This result does not strictly apply to the actual vapour pressure e, only to the equilibrium vapour pressure e_{eq}. The two are linked through the relative humidity ($\equiv e/e_{eq}$ –see Chapter 1). It is an important result from climate models that they show only weak changes in relative humidity with anthropogenic forcing, thereby implying that de/e also follows the Clausius-Clapeyron scaling.

Q4. This statement is wrong. First, as expressed by the thermal wind relation, horizontal temperature gradients drive the vertical gradient of the wind, not the wind itself. For example, applying the relation in **Q3**(i) (Chapter 4),

$$u_2 - u_1 \approx \frac{R_d}{f}\left(\ln\frac{P_2}{P_1}\right)\frac{\partial \overline{T}}{\partial y} \tag{2}$$

where level 1 is taken as that where the jet is strongest (tropopause) and level 2 is taken as the Earth's surface. This shows that changes in $\partial\overline{T}/\partial y$ will affect $u_2 - u_1$, not u_1.

Let's nevertheless assume that one could ignore the latter (maybe because they could be small compared to those occurring higher up) and estimate by how much one would change the jetstream velocity u_1. From the formula,

$$\delta u_1 \approx -\frac{R_d}{f}\left(\ln\frac{P_2}{P_1}\right)\delta\left(\frac{\partial \overline{T}}{\partial y}\right) \tag{3}$$

in which δ indicates the change as a result of increasing greenhouse gas concentrations. Indeed, if the poles warm up quicker than the equator, $\delta(\partial\overline{T}/\partial y) > 0$ (remember that $\partial T/\partial y < 0$ since temperature decreases poleward in the troposphere) and so the jet weakens ($\delta u_1 < 0$). The larger warming ($\simeq 1K$) of the Arctic is confined to a surface layer extending from

$1000hPa$ to about $600hPa$. So, the vertically averaged change in temperature is weighted by a factor $600/1000$ compared to that occurring in this layer. Taking the gradient from equator to pole, f at $45°N$, $P_1 = 200hPa$ (tropopause pressure) at that latitude, $P_2 = 1000hPa$, one degree of latitude $\approx 100km$, we get,

$$\delta u_1 \approx -\frac{600}{1000} \times \frac{287}{10^{-4}} \times \left(\ln\frac{1000}{200}\right)\left(\frac{1K}{90 \times 100km}\right) \approx -0.3ms^{-1} \quad (4)$$

In reality, the change would be even smaller because the equator-to-pole temperature gradient actually increases at upper levels (we'll see this when we talk about climate change). Considering the average jetstream speed is $\approx 30ms^{-1}$, the change caused by Arctic warming is negligible.

NB: A word of caution. The vertical shear of the wind seems indeed not to be changed too much by Arctic warming. But it might be that the wind itself decreases more substantially as a result of Arctic warming (the latter might perturb the path of the storms and the excitation of Rossby waves, possibly leading to less momentum transport into the jetstream). We would need a climate model to look into this interesting possibility...

Q5.

(i) The timescale for the upper ocean is $t_u = \rho_o c_o h/\alpha$ and that for the deep ocean $t_d = \rho_o c_o H/\alpha$. Using the values given, one obtains $t_u \approx 4yrs$ and $t_d \approx 165yrs$. For timescales $\gg t_u$, upper ocean temperature anomalies have had time to equilibrate with the forcing so one can consider the upper ocean to have zero heat capacity. On timescales $\ll t_d$, deep ocean temperature anomaly have not had time to develop and so one can treat the deep ocean as a heat reservoir.

(ii) On short timescales ($\ll t_u$) after the forcing has been switched on, the upper ocean hasn't had time to develop a significant temperature anomaly, i.e., $T' \approx 0$. The imbalance at the TOA is thus simpy $N' \approx F' = 4Wm^{-2}$.

(iii) On intermediate timescales ($\gg t_u$ but $\ll t_d$), the upper ocean has had time to develop a temperature anomaly in response to anthropogenic heating. At these timescales, the upper ocean is in equilibrium with this forcing F' and the heat transported to the deeper ocean O': $N' = O'$. Using $O' = \kappa T'$, this leads to $T' = F'/(\alpha + \kappa)$ and $N' = F'\kappa/(\alpha + \kappa)$. Putting numbers, one obtains $T' \approx 1.9K$ and $N' = 0.95Wm^{-2}$.

(iii) At equilibrium there is not net energy imbalance at the TOA, thus $N' = 0$ and $T' = F'/\alpha = 2.5K$. This temperature change (equilibrium response to a doubling of CO_2) is called “climate sensitivity”.