

# Ertel's PV, gyres and the thermohaline circulation

(Notes for the BCCR "Bjerknes' lectures")

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## Contents

<b>1</b>	<b>Motivation</b>	<b>2</b>
<b>2</b>	<b>A simple view of PV</b>	<b>3</b>
2.1	PV as simply measuring the geometry of oceanic isopycnals . .	3
2.2	PV entry / exit as a simple problem of restratification and destratification . . . . .	5
<b>3</b>	<b>Computing PV entry / exit from observations</b>	<b>5</b>
3.1	Mechanical PV entry / exit . . . . .	5
3.2	Diabatic PV entry / exit . . . . .	6

<b>4</b>	<b>What do we learn?</b>	<b>9</b>
4.1	Winds vs. diabatic effects . . . . .	9
4.2	North Pacific stratification: gyres, freshwater flux and ocean eddies . . . . .	10
4.3	North Atlantic Thermohaline circulation . . . . .	13
<b>5</b>	<b>References</b>	<b>14</b>

# 1 Motivation

The purpose of these two lectures is to discuss a recent estimate of the distribution of oceanic potential vorticity (PV) sources and sinks at the sea surface (Czaja and Hausmann, 2008). This is important because PV is to a very good approximation a simple measure of the oceanic stratification (see below). By investigating the processes which set PV entry / exit at the sea surface we investigate the processes which control the large-scale stratification of the oceans. [NB: Why not study the density field itself then? PV introduces a simplification in that, unlike the volume of an isopycnal layer, it can not be changed by processes acting along an isopycnal surface like interior mixing; it can only change where the isopycnal layer intersects a boundary like the sea surface, the sea floor or the coasts –see Marshall and Nurser (1992). This simplifies PV budgets compared to density budgets].

The relationship between mixed layer processes and the oceanic interior has been studied considerably in theory and observations of the subduction process (Cushman-Roisin, 1987; Marshall and Nurser, 1992; Marshall et al., 1993). The sea surface entry - exit of PV has however never been mapped from observations.

The two lectures are ‘self contained’ and start from scratch. In the first lecture, I will give some background on PV, a simple physical interpretation of the ‘entry/exit of PV at the sea surface’, and discuss how the latter can be

estimated from observations (sections 2 and 3 below). In the second lecture I will focus on the most interesting features provided by the maps (section 4 below). These notes only highlight the key results –see the lecture slides for more.

## 2 A simple view of PV

The major simplification in what follows arises from the fast rotation of the planet (rotation rate  $\Omega$ ) compared to the local spin of oceanic motions (characterized by velocity  $U$  and horizontal scale  $L$ ), i.e.,

$$R_o = \frac{U}{\Omega L} \ll 1 \quad (\text{small Rossby number}) \quad (1)$$

[NB: This result also applies to the energetic ocean eddies: for  $U = 50 \text{ cm/s}$  and  $L = 100 \text{ km}$ , the Rossby number is still small,  $R_o \simeq 0.1$ . This would not be true of their atmospheric analogs.]

As a result, the projection of the vorticity vector  $\vec{\eta}$  onto the isopycnal normal  $\vec{\nabla}\sigma$ , i.e., Ertel's PV,

$$PV \equiv -\frac{\vec{\eta} \cdot \vec{\nabla}\sigma}{\rho} \quad (2)$$

is well approximated by retaining only the planetary contribution  $2\vec{\Omega}$  to  $\vec{\eta}$ ,

$$PV \simeq -\frac{2\vec{\Omega} \cdot \vec{\nabla}\sigma}{\rho_o} \quad (3)$$

in which we have further used the weak compressibility of water ( $\rho \simeq \rho_o$ , a constant). Two major consequences of this are now discussed.

### 2.1 PV as simply measuring the geometry of oceanic isopycnals

Isopycnals are nearly vertical in the mixed layer (vertical turbulent processes dominate over horizontal geostrophic effects) but nearly horizontal in the interior (horizontal geostrophic effects dominate over vertical turbulent effects).

As a result, (3) becomes simply

$$PV_{ml} \simeq \frac{2\Omega \cos \phi}{\rho_o} \times \frac{\partial \sigma_{ml}}{\partial y} \quad (4)$$

$$PV_{int} \simeq -\frac{2\Omega \sin \phi}{\rho_o} \times \frac{\partial \sigma_{ml}}{\partial z} \quad (5)$$

The ratio  $PV_{ml}/PV_{int}$  is proportional to the isopycnal slope (Fig. 1),

$$\tan \alpha \equiv \frac{\partial \sigma_{ml}}{\partial y} / \frac{\partial \sigma_{ml}}{\partial z} = \frac{\Delta z}{\Delta y} \ll 1 \quad (6)$$

so that the interior PV is much larger than that of the mixed layer. The oceanic flow can effectively stir the density field so that horizontal PV gradients do not necessarily reflect the simple latitudinal dependences seen in the approximate equations for  $PV_{ml}$  and  $PV_{int}$  above.

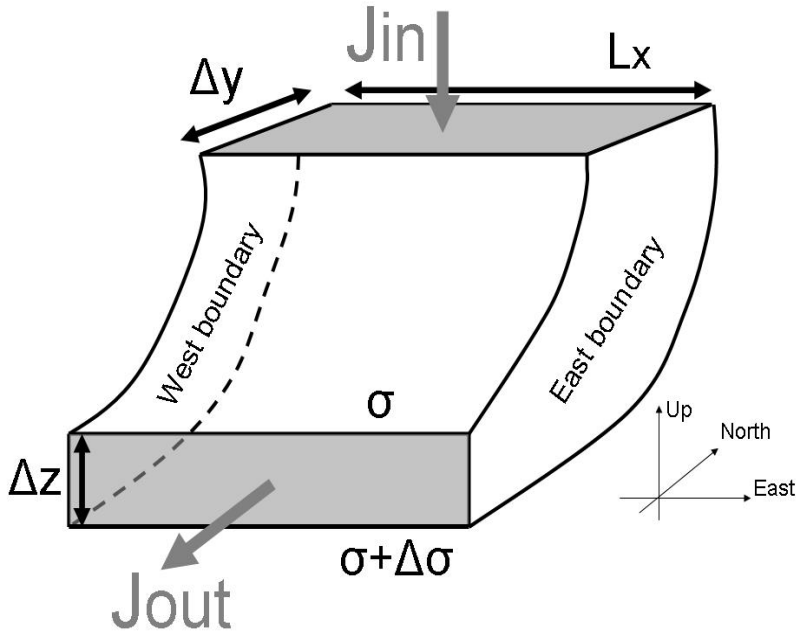


Figure 1. Geometry of an isopycnal layer.

## 2.2 PV entry / exit as a simple problem of restratification and destratification

Surface buoyancy gain or Ekman drift (wind forcing) from light to dense tilt horizontally the isopycnals. This brings the isopycnals' normal closer to the planet's axis of rotation, hence increase the PV (surface PV entry). Conversely, a surface buoyancy loss or an Ekman drift from dense to light tilt vertically the isopycnals, bringing their normal away from the planet's axis of rotation: PV decreases (surface PV exit). [NB: a more general view, based on momentum budget considerations, is given in Czaja and Hausmann (2008) –see also Thomas (2005)].

This effect is felt 'locally' (i.e., in the physical  $x, y, z$  space). It is also leading to an increase of PV for the isopycnal layer considered (i.e., in the  $x, y, \sigma$  space). Averaging over many of these 'entry' or 'exit' events, the isopycnal layer, as a whole, must remove this excess PV (or fill-in this PV deficit). A classic example of such a problem is Stommel's explanation of the western boundary currents (1948): PV exit driven by winds opposed by PV entry driven by frictional effects at the western boundary.

Finally, it is worth mentioning another important implication of the eq. (3): any PV budget can be rephrased in terms of buoyancy budget.

## 3 Computing PV entry / exit from observations

### 3.1 Mechanical PV entry / exit

The wind PV entry / exit is formally given by (e.g., Marshall and Nurser, 1992)

$$J_s^{(mech)} \equiv (\vec{F} \times \vec{\nabla} \sigma)_{z=0} \cdot \vec{k} \quad (7)$$

in which  $\vec{F}$  represents the non conservative forces in the momentum budget. For practical purposes, we consider the simplification

$$J_s^{(mech)} \simeq \left( \frac{\vec{\tau}_s}{\rho_o h} \times \vec{\nabla} \sigma_m \right) \cdot \vec{k} \quad (8)$$

which can be computed from a climatology of surface windstress  $\vec{\tau}_s$ , mixed layer density  $\sigma_m$  and depth  $h$ . Note the clear connection of Eq. (8) to the Ekman drift discussed in section 2.2.

Figure 2 displays the result of the calculation for the Northern Hemisphere. Large - scale features dominate the plot, with PV exit poleward of about  $30^\circ N$  associated with westerlies and equatorward of the ITCZ (dense to light Ekman drift). PV entry is seen poleward of the ITCZ and equatorward of about  $30^\circ N$  (light to dense Ekman drift). An isopycnal calculation somewhat simplifies the picture: PV exit over dense waters, PV entry for light waters (Fig. 3).

### 3.2 Diabatic PV entry / exit

The diabatic PV entry / exit is formally given by (Marshall and Nurser, 1992)

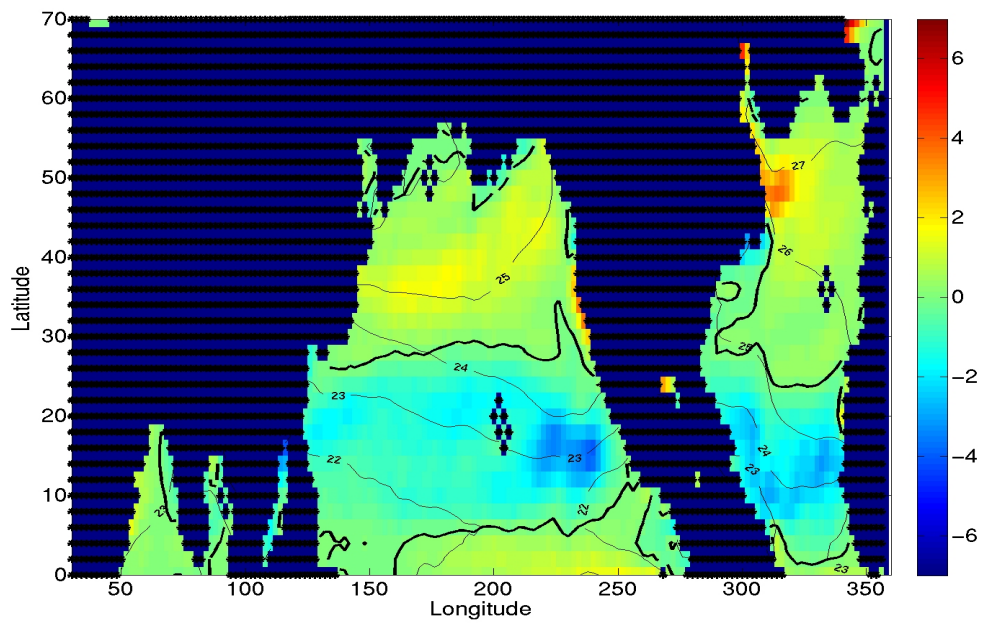
$$J_s^{diab} = \left( \frac{D\sigma}{Dt} \vec{\eta} \right)_{z=0} \cdot \vec{k} \quad (9)$$

and is approximated using a slab mixed layer model, i.e.,

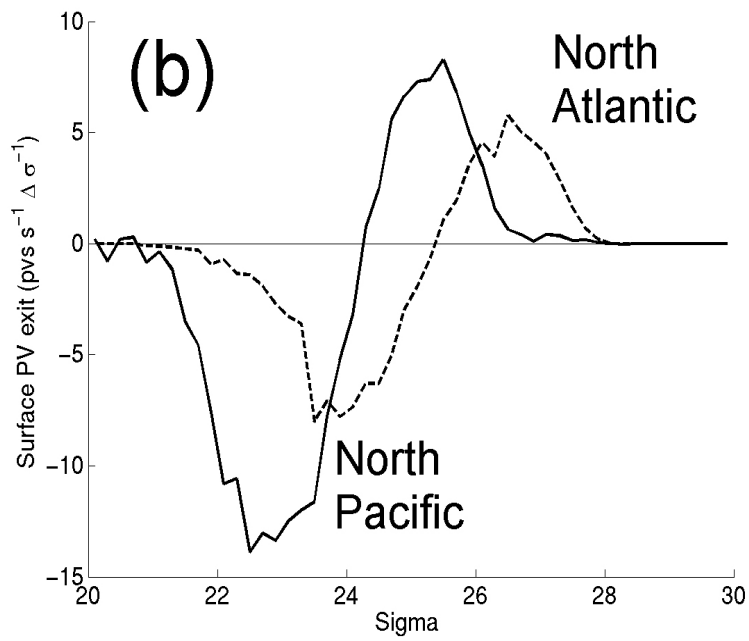
$$J_s^{diab} = \frac{f D_{air-sea}}{h} + \frac{f D_{ent}}{h} + \frac{f D_{eddy}}{h} \quad (10)$$

in which  $D_{air-sea}$  is the surface buoyancy flux,  $D_{ent}$  the mixed layer buoyancy loss through entrainment and  $D_{eddy}$  the effect of ageostrophic eddies on the mixed layer. The latter term can not be estimated on the global scale from data alone and has not been taken into account below. The effects of entrainment are parameterized.

Figure 4 displays the result of an isopycnal calculation for the North Atlantic and Pacific. PV exit at high density is seen in the Atlantic and the



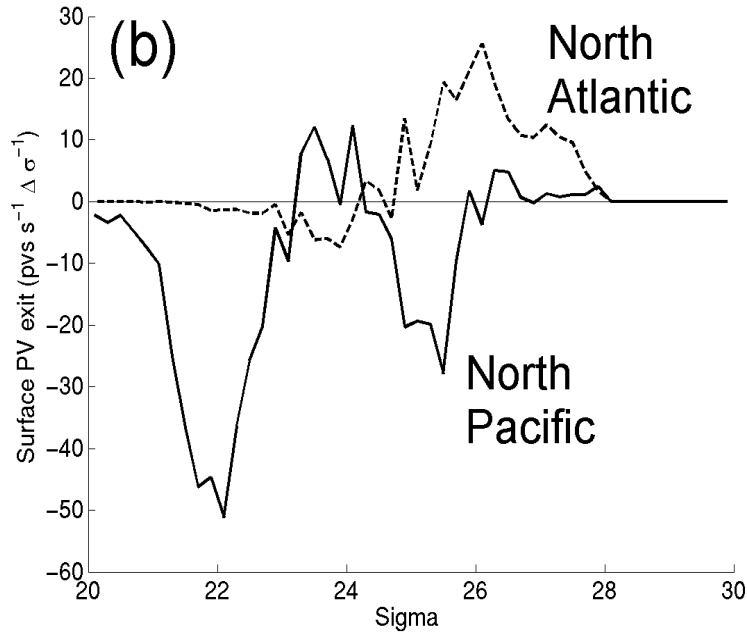
**Figure 2.** Annual mean PV exit (red) and entry (blue) driven by surface winds at fixed location (in units of  $10^{-12} \text{ pvs } m^2 \text{ s}^{-1}$ ). The annual mean position of surface isopycnals is given in light black contours. See Czaja and Hausmann (2008) for more details on the calculation.



**Figure 3.** Annual mean PV exit (positive values) and entry (negative values) driven by surface winds following isopycnal layers of  $\Delta\sigma = 0.2 \text{ kg m}^{-3}$  bin width. See Czaja and Hausmann (2008) for more details on the calculation.



Pacific. Intense surface cooling over the separated Gulf Stream and Kuroshio are seen here as moderate PV exit over intermediate density classes ( $\sigma \simeq 25$  and  $\sigma \simeq 24$ , respectively). At low density, both basins show PV entry, a feature particularly pronounced in the Pacific. The PV entry at  $\sigma \simeq 25$  in the Pacific is discussed further below.



**Figure 4.** Annual mean PV exit (positive values) and entry (negative values) driven by surface buoyancy and entrainment flux, following isopycnal layers of  $\Delta\sigma = 0.2 \text{ kg m}^{-3}$  bin width. See Czaja and Hausmann (2008) for more details on the calculation.

## 4 What do we learn?

### 4.1 Winds vs. diabatic effects

Figure 5 displays the Northern Hemisphere isopycnal calculation for mechanical (blue) and diabatic (red) PV entry / exit. At high and low densities, diabatic effects dominate. This is somewhat expected for the high density

end considering the emphasis of the literature on the driving of thermohaline circulation by cooling at high latitudes. It is somewhat more surprising at low densities, considering the focus on adiabatic dynamics in models of El Nino. Hence, Figure 5 reminds us that diabatic effects are key in setting the stratification in the Tropics <sup>1</sup>. (Further analysis indicates this is mostly reflecting the buoyancy gain through heating rather than precipitation).

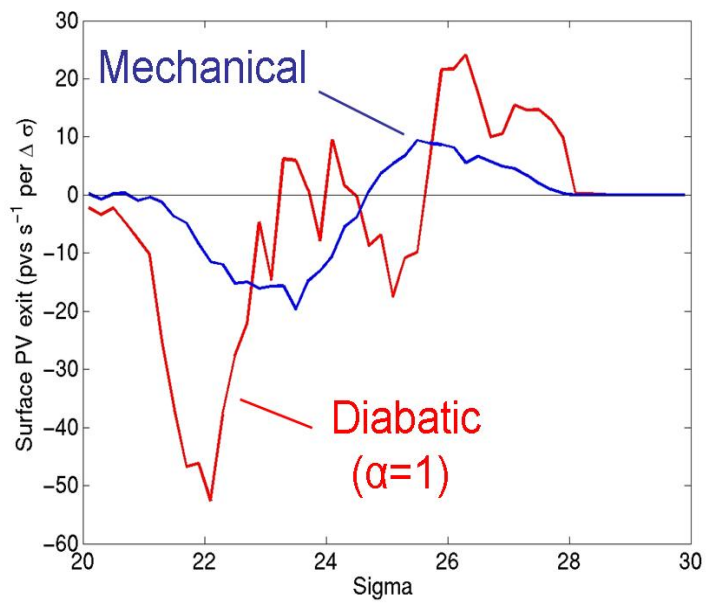
At intermediate densities ( $23 \leq \sigma \leq 26$ ), mechanical and diabatic effects are comparable and oppose each other. Some of this is easy to understand: in the subtropics strong surface evaporation leads to PV exit but the light to dense Ekman drift tends to restratify. This is in agreement with the analysis of Marshall et al. (1993) who argued for a key role of Ekman heat transport in providing the heating required for subduction. The PV diabatic entry and mechanical PV exit seen at  $\sigma \simeq 25$  is more surprising and comes from the North Pacific (Fig. 4). We discuss this next.

## 4.2 North Pacific stratification: gyres, freshwater flux and ocean eddies

Further inspection of the isopycnal locations show that in the North Pacific,  $\sigma \simeq 25$  corresponds to the boundary between the subtropical and subpolar gyres. This density class is flanked by PV exit on the higher density side (wintertime cooling driven PV exit –the  $\sigma = 27$  does not outcrop in the annual mean), and on the lower density side (Kuroshio cooling), as displayed in Fig. 6. This intergyre PV entry is driven by buoyancy gain, more precisely by the summertime freshening experienced by the  $\sigma \simeq 25$  density class, a remote effect of the strong Asian monsoon (Emile-Geay et al., 1998; Czaja, 2008). A simple way to see this is to plot the Sverdrup streamfunction superimposed on the annual ( $E - P$ ) (Fig. 7). Unlike in the Atlantic where the intergyre boundary coincides with the zero ( $E - P$ ) line, these lines are

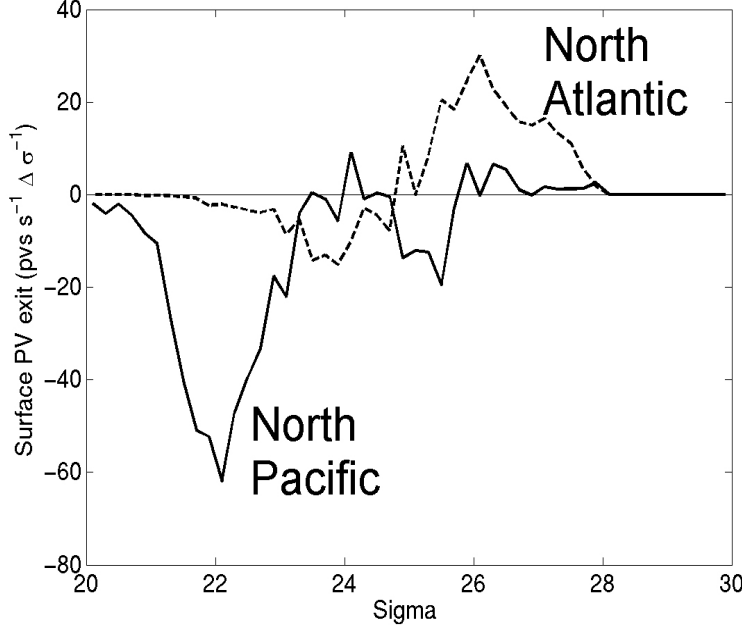
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<sup>1</sup>The ITCZ corresponds roughly to  $\sigma = 22$ .



**Figure 5.** Same as Fig. 4, but for the Northern Hemisphere (i.e., Atlantic + Pacific + Indian) PV exit / entry associated with mechanical and diabatic forcings.

shifted in the North Pacific.



**Figure 6.** Same as Fig. 4 but for the net PV entry / exit.

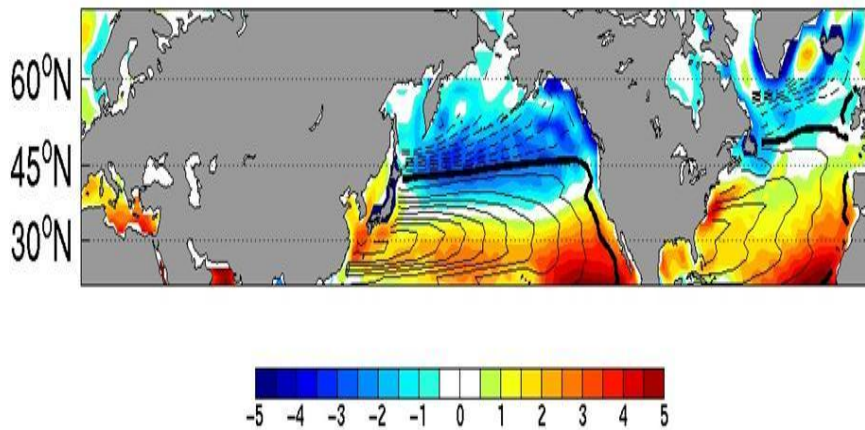
At the inter-gyre boundary, the Sverdrup transport goes to zero and there is little mean northward flow in the upper layers of the North Pacific. It is then conceivable that the bulk of the PV transport below the mixed layer is due to eddy fluxes. To match a sea surface PV entry, the interior eddy PV flux must be equatorward, that is downgradient. This is plausible. Let's now check the order of magnitude. Using the impermeability property (Fig. 1,  $\int J_{in}dS = \int J_{out}dS$ ), this can happen if,

$$J_{in} \times L_x \times \Delta y = J_{out} \times L_x \times \Delta z \simeq \rho_o \overline{V'Q'} \times L_x \times \Delta z \quad (11)$$

or, using  $\rho_o \overline{V'Q'} = \rho_o V'(\mu \overline{Q}) \simeq \mu f V' \Delta \sigma / \Delta z$ , in which  $\mu$  scales (in percent) the PV fluctuation,

$$V' \simeq \frac{J_{in}}{f\mu} \times \frac{\Delta y}{\Delta \sigma} \quad (12)$$

For  $\mu = 0.1$  (i.e., 10 % fluctuations in PV),  $J_{in} = 10^{-12} \text{ pvs } m^{-2} s^{-1}$  (typical amplitude of PV fluxes) and  $\Delta y / \Delta \sigma = 1000 \text{ km} / 1 \text{ kg } m^{-3}$ , the required velocity perturbation is  $10 \text{ cm} / \text{s}$ , not implausible [NB: this estimates assumes



**Figure 7.** Annual mean (E-P) in units of  $10^{-5}kg s^{-1} m^{-2}$  superimposed on the Sverdrup streamfunction (CI = 10Sv). All calculations made with NCEP-NCAR reanalyses data –see Czaja (2008) for details.

perfectly correlated velocity and PV fluctuations extending over the whole zonal length of the density layer].

Open question: how would this work in a coarse ocean model? A possibility is that the zonal deceleration of the model’s Kuroshio by friction acts in effect as an equatorward and downward PV transport, in a dynamics akin to that of the ‘eddy free’ subpolar gyre model of D. Marshall (2000).

### 4.3 North Atlantic Thermohaline circulation

The intergyre PV entry is not seen in the North Atlantic although there are also strong ocean eddies and large precipitation in the atmospheric storm track there! This is because the whole North Atlantic pattern is dominated by surface (evaporative) cooling and its associated PV exit.

Considering that the surface cooling is driven by the North Atlantic Drift, this suggests a PV-based model of the thermohaline circulation. The circulation drives surface PV exit over dense layers. The latter must, in steady state, replenish their PV somehow. This can happen at the sea surface in the Southern Ocean, or where these isopycnals intersect the ocean bottom

(or coastlines). Observations do not rule out the air-sea interaction route –in essence that proposed by Toggweiler and Samuels (1995). Indeed to match the PV exit  $J_{out}$  driven by surface cooling  $J_{out} \simeq fD_{in}/h$ , the mechanical PV entry over the Southern ocean  $J_{in} \simeq \frac{\tau_x}{\rho_0 h} \frac{\partial \sigma_m}{\partial y}$  must satisfy (Fig. 1),

$$J_{in} \Delta y L_x^{SO} = J_{out} \Delta y L_x^{NATL} \quad (13)$$

i.e., introducing a surface cooling  $Q_{in}$  in  $Wm^{-2}$  and a typical meridional temperature gradient  $T_{m,y}$ ,

$$Q_{in} = C_p \frac{\tau_x T_{m,y}}{f} \times \frac{L_x^{SO}}{L_x^{NATL}} \quad (14)$$

For a ratio of zonal scales  $\simeq 3$ ,  $T_{m,y} = 1K/100km$ ,  $\tau_x = 0.1Nm^{-2}$ , one obtains  $Q_{in} \simeq 100 Wm^{-2}$ , a reasonable number.

This PV view of the thermohaline circulation does not emphasize the effects of internal mixing as most of the literature does (e.g., Wunsch and Ferrari, 2004). This is because mixing effects are ‘hidden’ in this analysis: internal mixing can not change the PV of an isopycnal layer but it will change its location. In particular, it can control whether or not a deep isopycnal layer intersects a topographic feature, thereby altering the frictional PV sources and sinks of this layer.

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