

The Global Atmospheric Circulation in Moist Isentropic Coordinates

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ABSTRACT

Differential heating of the earth's atmosphere drives a global circulation that transports energy from the tropical regions to higher latitudes. Because of the turbulent nature of the flow, any description of a "mean circulation" or "mean parcel trajectories" is tied to the specific averaging method and coordinate system. In this paper, the NCEP–NCAR reanalysis data spanning 1970–2004 are used to compare the mean circulation obtained by averaging the flow on surfaces of constant liquid water potential temperature, or dry isentropes, and on surfaces of constant equivalent potential temperature, or moist isentropes. While the two circulations are qualitatively similar, they differ in intensity. In the tropics, the total mass transport on dry isentropes is larger than the circulation on moist isentropes. In contrast, in midlatitudes, the total mass transport on moist isentropes is between 1.5 and 3 times larger than the mass transport on dry isentropes.

It is shown here that the differences between the two circulations can be explained by the atmospheric transport of water vapor. In particular, the enhanced mass transport on moist isentropes corresponds to a poleward flow of warm moist air near the earth's surface in midlatitudes. This low-level poleward flow does not appear in the zonally averaged circulation on dry isentropes, as it is hidden by the presence of a larger equatorward flow of drier air at same potential temperature. However, as the equivalent potential temperature in this low-level poleward flow is close to the potential temperature of the air near the tropopause, it is included in the total circulation on moist isentropes. In the tropics, the situation is reversed: the Hadley circulation transports warm moist air toward the equator, and in the opposite direction to the flow at upper levels, and the circulation on dry isentropes is larger than that on moist isentropes.

The relationship between circulation and entropy transport is also analyzed. A gross stratification is defined as the ratio of the entropy transport to the net transport on isentropic surfaces. It is found that in midlatitudes the gross stability for moist entropy is approximately the same as that for dry entropy. The gross stratification in the midlatitude circulation differs from what one would expect for either an overturning circulation or horizontal mixing; rather, it confirms that warm moist subtropical air ascends into the upper troposphere within the storm tracks.

1. Introduction

The global atmospheric circulation redistributes energy and entropy from equatorial regions to higher latitudes. This is accomplished by a combination of a poleward flow

of high energy and high entropy air parcels and an equatorward return flow of parcels with lower energy and entropy content. Because of the turbulent nature of the atmosphere, individual parcel trajectories vary widely, and the circulation can only be described in an averaged sense. A key issue arises from the fact that different averaging methods can produce different outcomes.

The Eulerian-mean circulation is one of the better known descriptions of the circulation, and is obtained by averaging the flow at constant pressure or geopotential

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height. Eulerian-mean circulations exhibit a classic three-cell structure: a direct Hadley circulation in the tropics, an indirect Ferrel cell in midlatitudes, and a weak direct polar cell at high latitudes (see Peixoto and Oort 1992, among others). The circulation in Ferrel cells is toward the equator in the upper troposphere and toward the pole near the surface; it is associated with a net energy transport toward the equator. In the midlatitudes, however, the atmospheric energy transport remains toward the poles, owing to a large poleward transport by the eddies.

Alternatives to the Eulerian-mean circulation have been developed in an attempt to capture the contribution of midlatitudes eddies. The transformed Eulerian-mean circulation (Andrews and McIntyre 1976; Dunkerton 1978; Edmon et al. 1980; Iwasaki 1989; Jukes 2001) obtains a residual circulation by including a correction term that accounts for the mass transport by eddies, an approach similar to the Stokes drift (Stokes 1847) in shallow-water waves. This residual circulation differs markedly from the Eulerian-mean circulation in that it exhibits a single cell with air rising in the equatorial regions and subsiding over the poles, although there is also some evidence of secondary ascent in the midlatitudes (Edmon et al. 1980). The use of entropy as a vertical coordinate offers another—perhaps more direct—way to assess parcels' trajectories. In the atmosphere, various authors (Townsend and Johnson 1985; Johnson 1989; Jukes et al. 1994; Held and Schneider 1999; Schneider 2004; Schneider et al. 2006) have analyzed the circulation using either entropy or potential temperature as the vertical coordinates instead of pressure. McIntosh and McDougall (1996) follow a similar approach for the ocean based on potential density. Atmospheric and oceanic eddies are associated with large fluctuations of pressure and geopotential height. However, as long as the eddies are almost adiabatic, fluctuations in the parcels' entropy, potential temperature, or potential density are comparatively smaller. Averaging the circulation on isentropic surfaces provides a more direct description of the Lagrangian circulation, although some differences persist (Bowman and Carrie 2002). As for the transformed Eulerian-mean circulation, the circulation on isentropic surfaces exhibits a single global overturning cell from the equator to polar regions.

There is, however, an important problem with such analysis of the circulation on isentropic coordinates: isentropic surfaces in a moist atmosphere are not uniquely defined. This property is a consequence of the fact that the entropy of water vapor can only be defined up to an additive constant. This particular thermodynamic property of moist air is discussed in greater detail in the appendixes. In this paper, we compare the atmospheric circulation on two different sets of isentropic surfaces: surfaces of constant

liquid water potential temperature and surfaces of constant equivalent potential temperature. We will refer loosely to the former as "dry" isentropes and to the latter as "moist" isentropes. Pauluis et al. (2008) showed that the circulations on dry and moist isentropes differ markedly, with the mass transport on moist isentropes being approximately twice as large in the midlatitudes as the mass transport on dry isentropes. They also explained this difference by the presence in the midlatitudes of a large poleward flow of warm moist air at low level. The present paper further investigates these differences and shows how a joint analysis of the dry and moist circulation can provide new insights on the atmospheric circulation.

Daily National Centers for Environmental Prediction—National Center for Atmospheric Research (NCEP—NCAR) reanalysis data (Kalnay et al. 1996) from 1970 to 2004 are used to obtain the zonal-mean circulation using liquid water potential temperature and equivalent potential temperature as vertical coordinates. These results are discussed in section 2. Both analyses show a global overturning circulation with high potential temperature air flowing from the equatorial regions to the poles aloft and a return flow at lower potential temperature. They differ significantly, however, in the magnitude of the circulation. In the tropics, the total mass transport on dry isentropes is larger by approximately 20%–30% than the mass transport on moist isentropes. In contrast, in the midlatitudes, the total mass transport on moist isentropes is between 2 and 3 times larger than the transport on dry isentropes.

In section 3, we analyze the poleward mass transport in terms of the joint distribution of liquid water potential temperature and equivalent potential temperature. This approach allows us to better identify the properties of the moving air masses. In the tropics, it is found that, while the air parcels in the equatorward and poleward flow have similar values of equivalent potential temperature, their liquid water potential temperatures differ markedly. This situation is consistent with a direct overturning Hadley circulation and explains why the circulation on dry isentropes is larger than on moist isentropes. In midlatitudes, however, the additional mass flux on moist isentropes is due to the presence of a low-level poleward mass transport of warm moist air near the surface. The liquid water potential temperature in this poleward flow is characteristic of the lower troposphere, but its equivalent potential temperature is closer to the tropopause value. We argue that this low-level flow ascends into the upper troposphere in the storm tracks, resulting in enhanced precipitation there.

Section 4 discusses the entropy transport and the gross stratification in the circulation. The larger mass transport on equivalent potential temperature surfaces corresponds to enhanced moist entropy transport associated with the

poleward water vapor transport in midlatitudes. Surprisingly, it is also found that, in midlatitudes, the gross stratification for dry entropy and that for moist entropy are comparable. This situation stands in stark contrast to the tropics where the dry stratification is significantly larger than the gross moist stratification.

2. The global overturning circulation in isentropic coordinates

Thermodynamic entropy is a state variable that is used in Clausius formulation of the second law of thermodynamics. However, this thermodynamic entropy as used in atmospheric is not uniquely, an issue that is discussed in greater detail in appendices A and B. There are, arguably, two “sensible” choices for the entropy of moist air parcels: the moist entropy S_m and dry entropy S_l . These two entropies differ by a value proportional to the total water content. In most of this paper, we will follow the tradition of the atmospheric sciences and use potential temperature instead of entropy to define isentropic coordinates. The moist entropy S_m is related to the equivalent potential temperature θ_e by $S_m = C_p \ln(\theta_e/T_0)$. Similarly, the dry entropy S_l is related to the liquid water potential temperature θ_l by $S_l = C_p \ln(\theta_l/T_0)$; C_p is the heat capacity of air, and T_0 is an arbitrary reference temperature. Surfaces of constant liquid water potential temperature and surfaces of constant equivalent potential temperature define two independent sets of isentropic surfaces and are referred to here as dry and moist isentropes.

The liquid water potential temperature θ_l and equivalent potential temperature θ_e are two distinct state variables. In a moist atmosphere, the state of an air parcel can be characterized uniquely by a combination of these two potential temperatures and a third state variable, for example the total pressure p ; any state variable (such as temperature) can be expressed as a function of θ_e , θ_l , and p : $T = T(\theta_e, \theta_l, p)$. The two potential temperatures are conserved for reversible adiabatic processes, including reversible phase transitions. The key difference between these lies in how they change when water is either added or removed. The equivalent potential temperature is only slightly affected by the addition or removal of condensed water, but it increases greatly when water vapor is added to a parcel (for example from evaporation at the earth’s surface). In contrast, the addition or removal of water vapor does not change θ_l significantly, but the removal of condensed water by precipitation greatly increases it.

The zonal-mean circulation on both dry and moist isentropes is computed from the NCEP–NCAR reanalysis daily data for the period between January 1970 and December 2004. Starting from the gridded data on pressure surfaces, we compute θ_e and θ_l at each level. For the latter, as the NCEP–NCAR reanalysis does not provide the concentration of condensed water, we use the potential temperature. The data are then interpolated on 100 equally spaced pressure levels. The meridional mass flux $v\delta p$ is then boxed to obtain the mass transport $M(\theta_e, \theta_l, \phi) d\theta_e d\theta_l$ in terms of the dual distribution of θ_e and θ_l :

$$M(\theta_{e,0}, \theta_{l,0}, \phi) d\theta_{e,0} d\theta_{l,0} = \frac{1}{P} \int_0^P \int_0^{2\pi} \int_0^{p_{\text{sfc}}} [v\delta(\theta_e - \theta_{e,0})\delta(\theta_l - \theta_{l,0})a \cos\phi] \frac{dp}{g} d\lambda dt, \quad (1)$$

where P is the time period over which the data are averaged, a is the earth’s radius, ϕ is the latitude, g is the gravitational acceleration, and p_{sfc} is the surface pressure. When computing the mass flux numerically, the delta functions in (1) are replaced by summing the mass transport over boxes of finite width in θ_e and θ_l . The joint distribution for the monthly mean mass flux is computed based on a 35-year sample from the NCEP–NCAR data from 1970 to 2004. The streamfunctions on isentropic surfaces are obtained through the integrals:

$$\Psi_{\theta_e}(\theta_e, \phi) = \int_{-\infty}^{\theta_e} \left[\int_{-\infty}^{\infty} M(\theta'_e, \theta'_l, \phi) d\theta'_l \right] d\theta'_e, \quad (2a)$$

$$\Psi_{\theta_l}(\theta_l, \phi) = \int_{-\infty}^{\theta_l} \left[\int_{-\infty}^{\infty} M(\theta'_e, \theta'_l, \phi) d\theta'_e \right] d\theta'_l. \quad (2b)$$

Figure 1 shows the streamfunction on dry isentropes Ψ_{θ_l} (Figs. 1a,c,e) and moist isentropes Ψ_{θ_e} (Figs. 1b,d,f). In isentropic coordinates, the global circulation exhibits a single cell in each hemisphere. The circulation is direct with poleward flow of high entropy air and a return flow of low entropy air, and is associated with an equator-to-pole energy and entropy transports. Near the solstices, the center for the circulation is displaced into the summer hemisphere, and the intensity of the cross-equatorial cell increases significantly, while the cell confined within the summer hemisphere weakens.

The circulation on dry isentropes is, not surprisingly, similar to the analysis of the circulation using potential temperature obtained by Held and Schneider (1999). It shows a very strong overturning circulation from the equatorial regions all the way to the pole. This circulation

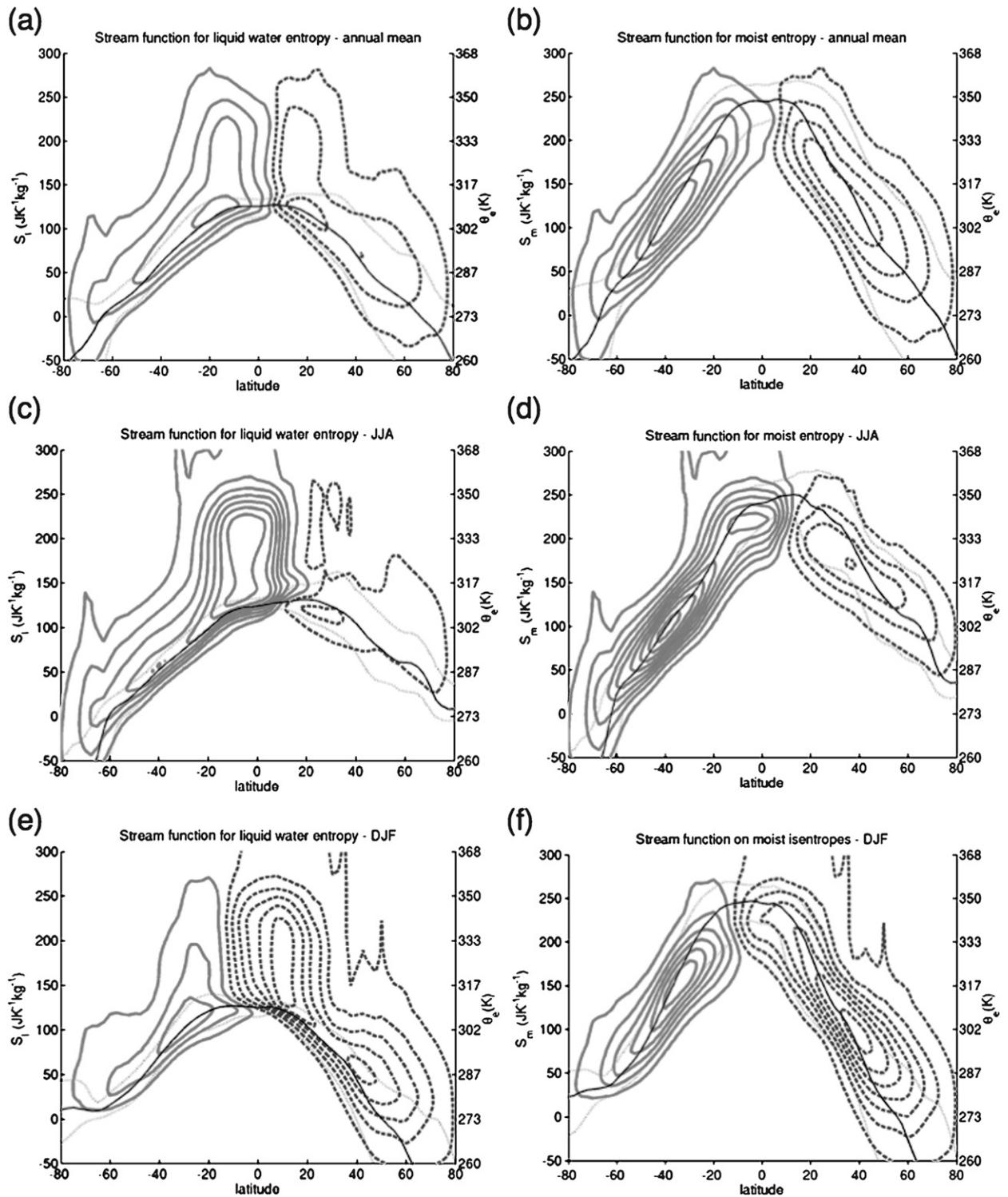


FIG. 1. Global circulation in isentropic coordinates. (a) Contour of annual mean streamfunction on dry isentropes Ψ_{θ_l} . The thick solid lines are for positive values corresponding to a southward flow at low θ_l and poleward flow at high θ_l ; thick dashed lines are for negative values corresponding to a circulation in the reverse direction. The thin lines show the 10th, 50th, and 90th percentile for surface θ_l . (b) As in (a), but for the streamfunction on moist isentropes Ψ_{θ_e} . (c),(e) As in (a), but for JJA and December–February (DJF); (d),(f) as in (b), but for JJA and DJF.

is strongest in the tropics and exhibits an intense cross-equatorial flow from the summer to the winter hemisphere, but it is comparatively weak in midlatitudes. Most notably, during June–August (JJA), the circulation almost disappears in Northern Hemisphere midlatitudes. The 10th, 50th, and 90th percentiles of the surface θ_l are depicted in black in Fig. 1. As noted in Juckes et al. (1994) and Held and Schneider (1999), the equatorward flow takes place on isentropes that intersect the surface.

Precipitation and surface sensible heat fluxes are the primary sources of θ_l in the atmosphere, while radiative cooling is its main sink. Their respective impact can be seen in Figs. 1a,c,e. Sensible heating from the surface appears through the increase in θ_l as air parcels move toward the equator. The strong “ascent” near the equator corresponds to latent heat release in the precipitating regions of the tropics. There is also some localized ascent in the midlatitudes, resulting from precipitation in the storm tracks. Radiative cooling shows up as the overall downward motions in the subtropics and at high latitudes.

The circulation on moist isentropes is similar to the analysis of the circulation based on the moist static energy performed by Czaja and Marshall (2006). The black lines in Figs. 1b,d,f depict the 10th, 50th, and 90th percentiles of the surface θ_e and span most of the vertical extent of the circulation. Fluctuations of θ_e near the surface are comparable to vertical variations across the troposphere. This is not surprising in the tropics where the deep convection maintains the temperature profile close to a moist adiabat (Xu and Emanuel 1989), but even in midlatitudes surface values of θ_e are close to their values in the upper troposphere.

Qualitatively, the circulation on moist isentropes exhibits the same single global cell structure as the circulation on dry isentropes, but there are significant quantitative differences. The circulation on moist isentropes exhibits a much weaker cross-equatorial flow and is generally weaker in the tropics. In contrast, it is strongest in the midlatitudes, with a maximum mass transport around 40° in both hemispheres. The circulation on moist isentropes is weaker in the summer hemisphere, albeit less so than for the circulation on dry isentropes. Most noticeably, while the circulation on dry isentropes almost disappears in the Northern Hemisphere during summer, the circulation on moist isentropes remains vigorous.

Evaporation and surface sensible heat fluxes are the main sources of θ_e , while radiative cooling is a sink. A key distinction between the dry and moist circulations lies in the different impacts that evaporation and condensation have on θ_l and θ_e . This results in different patterns of “rising” motions (in the sense of an increase of θ_l or θ_e) between the dry and moist circulations. The ascent in Figs. 1b,d,f spreads across the entire subtropics

and is broader than for the dry circulation. Regions of excess evaporation occupy a wider geographical area than regions of heavy precipitation. Note that this cross-isentropic ascent due to evaporation masks some of the radiative cooling in the subtropics, which is thus less apparent than in the circulation on dry isentropes. Second, the “subsidence” (i.e., decrease in θ_e or θ_l) poleward of 45° is significantly larger in the moist circulation than in the dry circulation. In the free troposphere, radiative cooling on a dry isentrope is partially balanced by latent heat release. In contrast, condensation and precipitation have little impact on θ_e , so that the change in θ_e in the free troposphere is due primarily to radiative cooling. Hence, the weaker subsidence in the dry analysis at high latitude must be attributed to a partial compensation between radiative cooling and condensation. Finally, the increase in θ_e in the return flow is much larger than the increase in θ_l because this increase reflects the increase in both temperature and latent heat content as air parcels move toward the equator.

The most striking difference between the dry and moist circulations lies in the overall mass transport. Figure 2 shows the mass transport in the overturning circulation, defined here as the difference between the maximum and the minimum of the streamfunction at any latitude:

$$\Delta\Psi_{\theta_l}(\phi) = \max_{\theta_l}[\Psi_{\theta_l}(\theta_l, \phi)] - \min_{\theta_l}[\Psi_{\theta_l}(\theta_l, \phi)], \quad (3a)$$

$$\Delta\Psi_{\theta_e}(\phi) = \max_{\theta_e}[\Psi_{\theta_e}(\theta_e, \phi)] - \min_{\theta_e}[\Psi_{\theta_e}(\theta_e, \phi)]. \quad (3b)$$

Both circulations are strongest in the winter hemisphere, but the circulations in the Southern Hemisphere are systematically more intense than those in the Northern Hemisphere for the same season. In the tropics, the mass transport on dry isentropes is larger by about 20%–30% than the circulation on moist isentropes. In contrast, in midlatitudes, the circulation on moist isentropes is larger than that on dry isentropes. The ratio of the midlatitude mass transport between the two analyses varies in the Northern Hemisphere between 1.5 during winter and 3 during summer; the ratio is 2 throughout the year in the Southern Hemisphere.

3. Mass transport in upper and lower branches of the circulation

In this section, we analyze why the circulation on moist isentropes is between 1.5 and 3 times larger than the circulation on dry isentropes in midlatitudes, while the circulation on dry isentropes is larger in the equatorial regions. This might seem surprising at first glance because, by averaging in isentropic coordinates, one aims to capture the mean motion of air parcels. If the isentropic

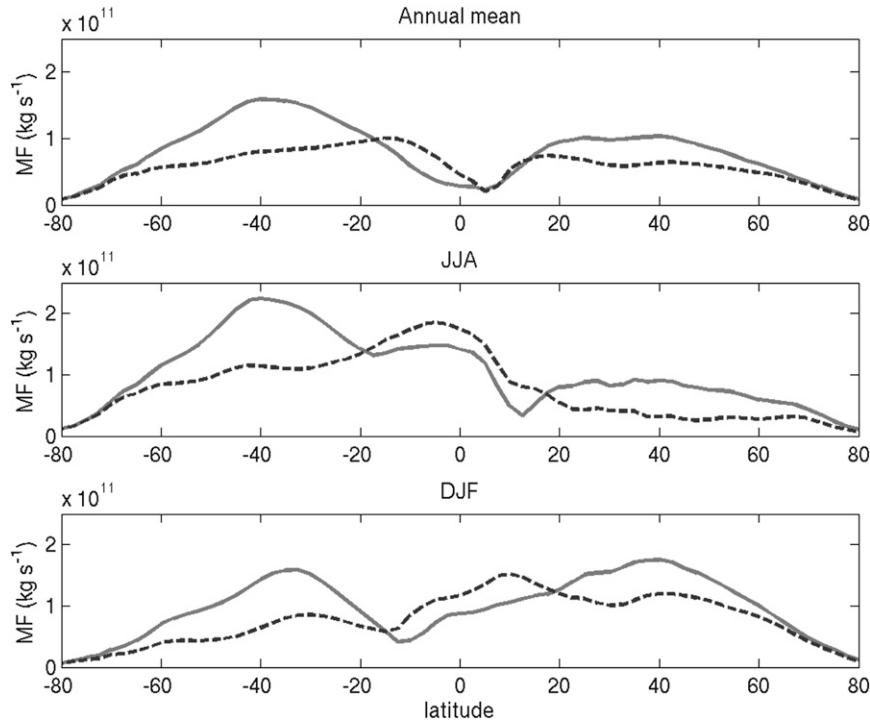


FIG. 2. Total mass transport on moist isentropes (solid lines) and dry isentropes (dashed lines) for the (top) annual mean, (middle) JJA, and (bottom) DJF circulation.

analysis exactly captured the Lagrangian mass transport, the total circulation in isentropic coordinates should be independent of any specific detail in the definition of entropy. As this is not the case, there must be some significant discrepancy between the mean parcel trajectory and either or both isentropic circulations. We argue that by averaging the circulation on dry isentropes one omits a significant portion of the midlatitude flow, while averaging on moist isentropes misses a portion of the circulation in the equatorial regions.

Pauluis et al. (2008) introduce here the concept of “isentropic filaments” as lines of constant value of θ_l and θ_e , that is, the intersection between a dry isentrope and a moist isentrope. As shown in the appendixes, definitions for the entropy of moist air differ from one another by a constant multiplied by the total water content. Any arbitrary definition of the entropy of moist air can, however, be expressed as a linear combination of the dry entropy and the moist entropy and is therefore constant along isentropic filaments. In contrast to the isentropic surfaces, the isentropic filaments do not depend on the specific choices made in the definition of the entropy. Any adiabatic invariant is conserved for reversible adiabatic transformations and is constant along isentropic filaments.

The global circulation can be described by analyzing the distribution of the meridional mass transport on isentropic filaments $M(\theta_l, \theta_e, \phi) d\theta_e d\theta_l$ at each latitude.

Figure 3 shows this joint distribution at 40° latitude in both hemispheres. As noted before, the circulation is direct, with high θ_e and high θ_l air parcels moving poleward and low θ_e and low θ_l returning equatorward. The total meridional mass transport on isentropic filament can be obtained by integrating the mass flow on all filaments with a net poleward transport:

$$\Delta\Psi_{\text{tot}} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H[\text{sign}(\phi)M(\theta_e, \theta_l, \phi)]M(\theta_e, \theta_l, \phi) d\theta_e d\theta_l. \quad (4)$$

Here $H(x)$ is the Heaviside function, with $H(x) = 1$ for $x \geq 0$ and $H(x) = 0$ otherwise. By definition, $\Delta\Psi_{\text{tot}}$ offers an upper bound on the magnitude of the streamfunction in any isentropic coordinate system.

When computing the streamfunctions on either dry or moist isentropes [Eqs. (2a), (2b)], we integrate the mass transport $M(\theta_e, \theta_l)$ on a half plane delimited by either a vertical (for Ψ_{θ_l}) or horizontal (for Ψ_{θ_e}) line. When computing the circulation on moist isentropes Ψ_{θ_e} , most of the air parcels at a given value of θ_e are moving in the same direction, that is, either poleward or equatorward. This implies that when the circulation is computed by integrating the mass transport M at constant θ_e there will be little cancellation between equatorward and poleward flows. In contrast, computing the mass transport

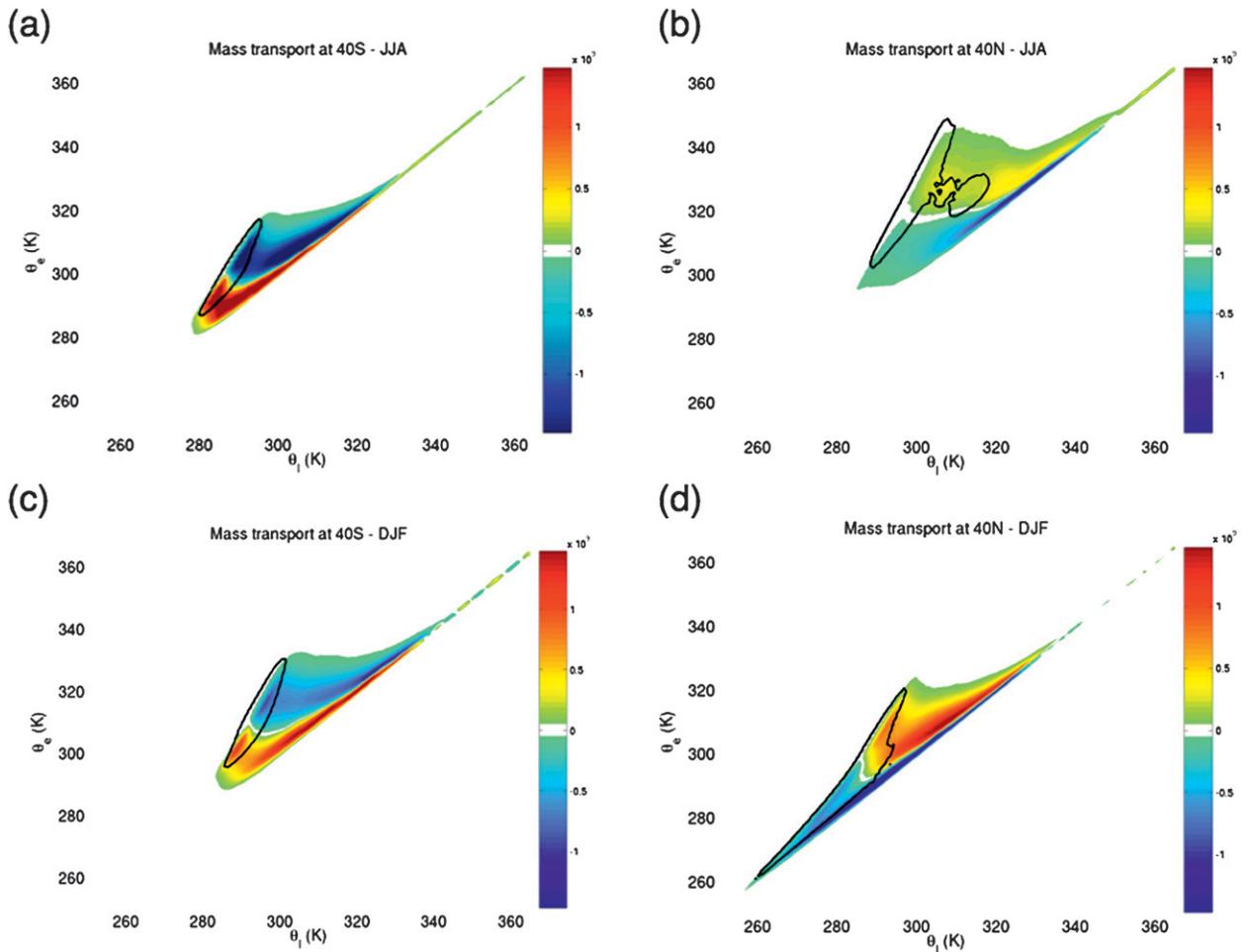


FIG. 3. Joint distributions of the mass transport as functions of dry entropy and moist entropy at (left) 40°S and (right) 40°N for (top) JJA and (bottom) DJF.

Ψ_{θ_l} on liquid water isentropes requires integration at a constant value of θ_l , which corresponds to a vertical line in Fig. 3. As there is often significant mass flux in both poleward and equatorward directions at a given value of θ_l , the net mass flux on dry isentropes corresponds to a small residual left after adding up the two opposite mass transports. Figure 3 indicates that poleward and equatorward parcels are better separated in terms of their equivalent potential temperature than in terms of their liquid water potential temperature. This fact alone implies that the circulation on moist isentropes should be larger than the circulation on dry isentropes.

In Fig. 3, the parcels with high θ_e and high θ_l are moving poleward and the parcels with low θ_e and low θ_l are moving equatorward. The difference in the meridional mass transport between the dry isentropes lies in that the parcels with high θ_e and low θ_l , corresponding to the isentropic filaments on the upper-left quadrant of the figure, are also moving poleward. (By definition $\theta_e > \theta_l$, and it is not possible to have a parcel with low θ_e and

high θ_l .) The mass transport on these isentropic filaments is accounted for in the global poleward mass transport at high θ_e but averaged out with the overall equatorward flow at low θ_l . The difference between the two potential temperatures (or corresponding entropies) is related to the total water content. Parcels with high θ_e and low θ_l have a high water content, and, given the atmospheric stratification for liquid water potential temperature, they must also be located near the surface. To confirm this, we show on Fig. 3 the 0.001 K^{-2} contour of the joint probability function for (θ_l, θ_e) at the earth's surface. This indicates the isentropic filaments that intersect the earth's surface. These near-surface filaments all have low values of θ_l , but can be subdivided between an equatorward flow of low θ_e air and a poleward flow of high θ_e air.

The equivalent potential temperature of the parcels in this near-surface poleward flow is very close to the liquid water potential temperature in the poleward flow in the upper troposphere. This is a very strong indication that these parcels are close to being convectively unstable

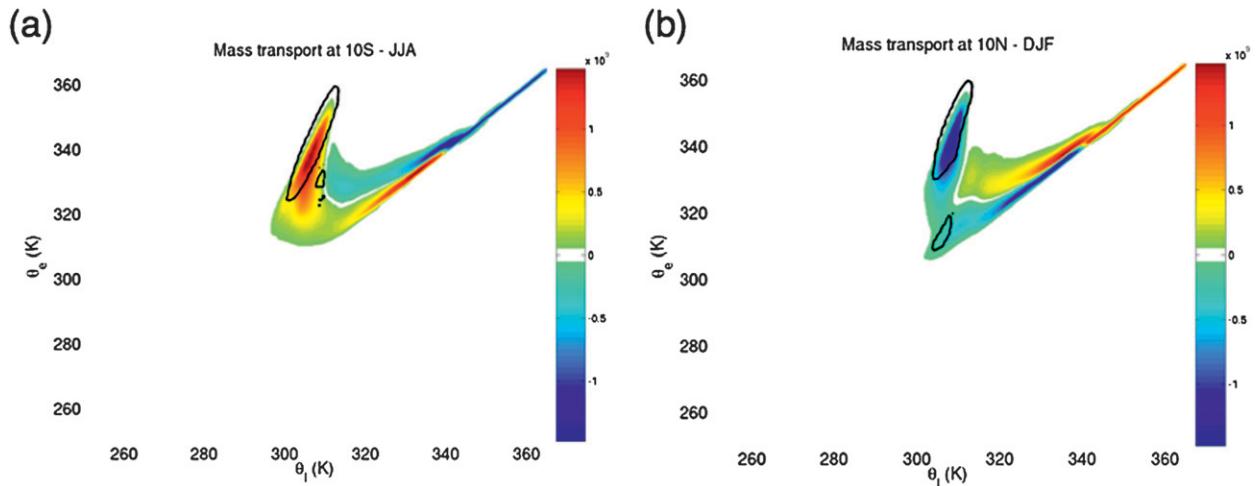


FIG. 4. Joint distributions of the mass transport as functions of the dry entropy and moist entropy at (a) 10°S in JJA and (b) 10°N in DJF.

and are ready to rise into the upper troposphere. These warm moist air parcels transport water from the subtropics into midlatitudes and dispose of it through precipitation as they ascend in the storm tracks. The mass transport in this low-level flow is comparable to the upper-tropospheric branch of the global circulation in midlatitudes, and one would thus expect that half of the air parcels near the tropopause in the polar region would have risen to this level within the midlatitude storm tracks rather than in equatorial regions.

The situation is opposite in equatorial regions, where total mass transport on moist isentropes $\Delta\Psi_{\theta_e}$ is smaller by about 30%–50% than the mass transport on liquid water isentropes. Figure 4 shows the joint distribution of the meridional mass transport on isentropic filaments $M(\theta_e, \theta_l)$ at 10°N during the northern winter and at 10°S during the southern winter—locations corresponding roughly to where mass transport is maximum in the Hadley cell. In contrast to midlatitudes, the equatorial distribution is characterized by an almost complete absence of low θ_e air. The poleward (winterward) and equatorward (summerward) moving air masses are much better separated by θ_l than by θ_e . On the one hand, when computing the mass transport Ψ_{θ_e} [Eq. (2a)], there is a strong cancellation between poleward and equatorward parcels. On the other hand, there is in much less cancellation when computing the circulation on dry isentropes Ψ_{θ_l} , which results in an enhanced circulation on dry isentropes. The distribution of the meridional mass transport on isentropic filaments $M(\theta_e, \theta_l)$ shown in Fig. 4 is typical of tropical regions. This is in agreement with the Hadley circulation, with a low-level flow of warm moist air with high θ_e and low θ_l toward the equator, and an upper-level poleward flow with high θ_e and high θ_l . One may also notice an equatorward flow of low θ_e air, with

$\theta_e < 320$ K and $\theta_l \sim 310$ K. This indicates relatively dry air parcels in the lower troposphere, but most likely above the boundary layer. This influx of dry, low θ_e air is balanced by the upper-tropospheric outflow of high θ_e , and thus contributes to a net export of energy out of the equatorial regions.

To illustrate the transition between the tropics and extratropics, we compute the mass transports $\Delta\Psi_1$, $\Delta\Psi_2$, and $\Delta\Psi_3$:

$$\Delta\Psi_1(\phi) = \int_{-\infty}^{305\text{K}} \left[\int_{-\infty}^{305\text{K}} M(\theta'_e, \theta'_l, \phi) d\theta'_l \right] d\theta'_e, \quad (5a)$$

$$\Delta\Psi_2(\phi) = \int_{-\infty}^{305\text{K}} \left[\int_{305\text{K}}^{-\infty} M(\theta'_e, \theta'_l, \phi) d\theta'_e \right] d\theta'_l, \quad (5b)$$

$$\Delta\Psi_3(\phi) = \int_{305\text{K}}^{\infty} \left[\int_{305\text{K}}^{\infty} M(\theta'_e, \theta'_l, \phi) d\theta'_l \right] d\theta'_e. \quad (5c)$$

These correspond to the transport associated respectively with the lower-left, upper-left, and upper-right quadrants of the mass flux distribution in Figs. 3 and 4 (as mentioned earlier, there is no parcel corresponding to the lower-right quadrant.). One can think of $\Delta\Psi_1$ as the low-level flow of dry air, $\Delta\Psi_2$ as the low-level flow of moist air, and $\Delta\Psi_3$ as the upper-level flow. The choice of the 305 K isentrope to define the different quadrants is somewhat arbitrary, but this is a typical value of a dry isentrope found within the middle troposphere in midlatitudes.

Figure 5 shows the mass transport in each of the three quadrants. The transport of low-level dry $\Delta\Psi_1$ air is always toward the equator, while the transport of upper-level $\Delta\Psi_2$ air is always poleward. The low-level flow of moist air $\Delta\Psi_2$, however, changes direction from equatorward in the tropics to poleward in midlatitudes. The streamfunction

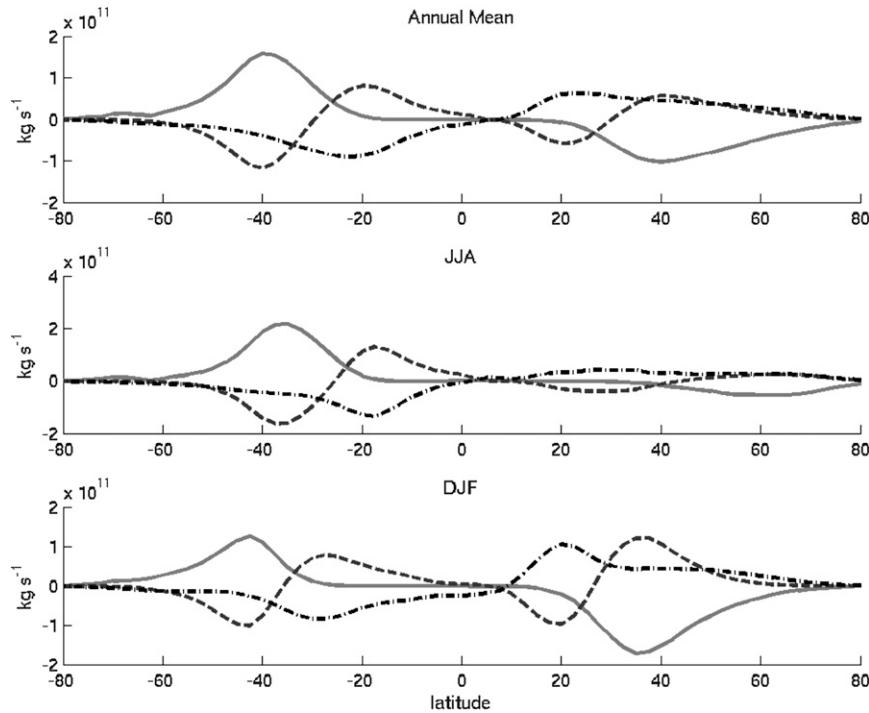


FIG. 5. Mass transport of low-level cold dry air $\Delta\Psi_1$ (solid line), low-level warm moist air $\Delta\Psi_2$ (dashed line), and upper-level warm dry air $\Delta\Psi_3$ (dash-dotted line) as defined in Eqs. (5a)–(5c): (top) annual; (middle) JJA; (bottom) DJF.

on dry isentropes is given by sum of the low-level mass transport of moist air and dry air: $\Psi_l(\theta_l = 305 \text{ K}, \phi) = \Delta\Psi_1(\phi) + \Delta\Psi_2(\phi) = -\Delta\Psi_3(\phi)$, while the streamfunction on moist isentropes is given by the low-level transport of dry air: $\Psi(\theta_e = 305 \text{ K}, \phi) = \Delta\Psi_1(\phi) = -\Delta\Psi_2(\phi) - \Delta\Psi_3(\phi)$. The difference between the moist and dry circulations can thus be related to the direction of the low-level flow of moist air. In the tropics, warm moist air moves in the same direction as the low-level flow of dry air, and the circulation on dry isentropes is larger than the circulation on moist isentropes. In contrast, in midlatitudes low-level warm moist air moves in the opposite direction to the low-level cold dry air, and the circulation on moist isentropes is larger than the circulation on dry isentropes. The circulation on moist isentropes usually becomes larger than the circulation on dry isentropes between 15° and 20° latitude. This corresponds to the transition between regions where the moisture transport is dominated by the Hadley circulation, in which the zonal-mean flow advects moisture toward the equator and regions in which eddies transport moisture poleward.

4. Entropy transport and gross stratification

The global atmospheric circulation is closely tied to an equator-to-pole entropy transport. The atmosphere

receives more energy than it emits in equatorial latitudes, while the emission of infrared radiation exceeds the energy inputs at high latitudes. The second law of thermodynamics implies that these external energy sources and sinks also act as entropy sources and sinks, with the net entropy input equal to the external energy source divided by the temperature at which it occurs. In addition, irreversible processes within the atmosphere produce a certain amount of entropy, as discussed in Pauluis and Held (2002). The ratio of this internal production to the external entropy sources scales as $\Delta T/T \sim 0.1$, where ΔT is the temperature difference between the energy sources and sinks, and T is a typical atmospheric temperature. The internal entropy production can be neglected to first approximation. One can thus think of the global atmospheric circulation as transporting entropy from the source regions in equatorial latitudes to entropy sinks at higher ones. Such global entropy transport is closely linked to the global energy transport discussed in Czaja and Marshall (2006).

For a given distribution of the meridional mass transport on isentropic filaments $M(\theta_l, \theta_e, \phi)$, one can compute the poleward entropy flux at each latitude:

$$F_{S_l}(\phi) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_l M(\theta_l, \theta_e, \phi) d\theta_l d\theta_e, \quad (6a)$$

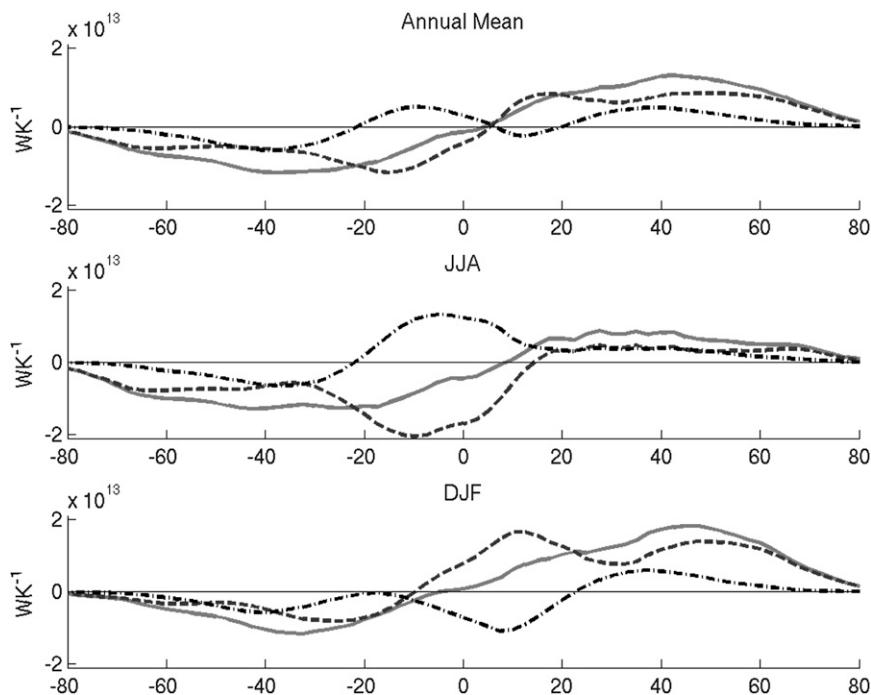


FIG. 6. Meridional entropy flux: moist entropy flux (solid line), dry entropy flux (dashed line), and contribution of the total water transport to the moist entropy flux (dash-dotted line): (top) annual mean, (middle) JJA mean, and (bottom) DJF mean.

$$F_{S_m}(\phi) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_m M(\theta_l, \theta_e, \phi) d\theta_l d\theta_e. \quad (6b)$$

Figure 6 shows the two entropy fluxes. Qualitatively, both show an equator-to-pole transport, broadly consistent with an entropy transport that is necessary to balance the earth's radiative forcing. They differ quantitatively: the dry entropy flux is larger than the moist entropy flux in low latitudes and smaller in midlatitudes. As discussed in the appendix B, radiative cooling, sensible heat fluxes, and irreversible processes have identical effects on the dry and moist entropies. However, the impacts of the hydrological cycle are quite different for dry entropy and moist entropy. On one hand, precipitation acts as a large source of dry entropy, but this has little effect on moist entropy. The intense precipitation in equatorial regions shows up as a large divergence in the dry entropy flux. The enhanced precipitation over the storm tracks is also a source of dry entropy, and its effect is most visible in the Northern Hemisphere winter. On the other hand, evaporation has little effect on dry entropy, but it is a source of moist entropy. The moist entropy source at the earth's surface is proportional to the total energy flux, that is, the sum of the sensible and latent heat fluxes. The moist entropy flux is divergent at low latitudes, where the atmosphere receives more energy than it loses, and

converges at high latitudes, where the atmospheric cooling exceeds the surface energy flux.

The difference between moist entropy and dry entropy is proportional to the liquid water content:

$$S_m - S_l = \delta s_0 q_T = \left[\frac{L_v(T_0)}{T} - R_v \ln \frac{e_0}{e_s(T_0)} \right] q_T,$$

where $L_v(T_0)$ is the latent heat of vaporization at the reference temperature T_0 . This implies that the difference between the moist and dry entropy fluxes is proportional to the total water transport:

$$F_{S_m} - F_{S_l} = \delta s_0 F_q, \quad (7)$$

where $F_q = \iint \overline{\rho v q_T} a \cos \phi d\lambda dz$ is the meridional transport of total water. The difference between the dry entropy and moist entropy fluxes in Fig. 6 is proportional to the total water transport. The Hadley circulation transports water vapor toward the equator, opposite to the global energy transport. Hence, at low latitudes, the dry entropy flux is larger than the moist entropy flux. In mid-latitudes, baroclinic eddies transport water vapor poleward, and the latent heat transport accounts for roughly half the total energy transport. Similarly, the water

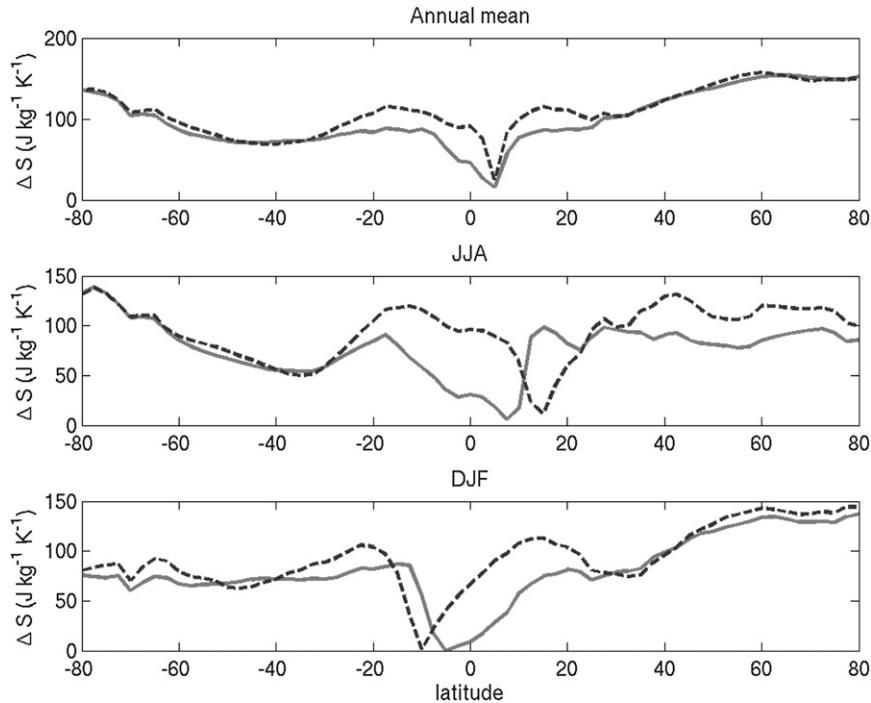


FIG. 7. Effective stratification for moist entropy (solid lines) and dry entropy (dashed lines) for the (top) annual mean, (middle) JJA, and (bottom) DJF circulations.

contribution accounts for about half the moist entropy transport in midlatitudes.

The relationship between mass and entropy transport can be assessed by defining an effective stratification as the ratio of the entropy flux to the total mass transport:

$$\Delta S_l = \frac{F_{S_l}}{\Delta \Psi_{\theta_l}}, \quad (8a)$$

$$\Delta S_m = \frac{F_{S_m}}{\Delta \Psi_{\theta_e}}. \quad (8b)$$

The effective stratification measures the ability of the circulation to transport entropy and is shown in Fig. 7. To understand the relationship between these two stratifications, one can first consider the vertical variations of moist entropy:

$$\frac{\partial S_m}{\partial z} = \frac{\partial S_l}{\partial z} + \delta s_0 \frac{\partial q_T}{\partial z}. \quad (9)$$

Dry entropy increases with height, but the water content decreases sharply. These two effects partially compensate each other, and the effective stratification for moist entropy should be smaller than the gross stratification for dry entropy.

Figure 7 shows that indeed the effective stratification for dry entropy in the equatorial regions is on the order

of $100 \text{ J K}^{-1} \text{ kg}^{-1}$ and is significantly larger than the gross stratification for the moist entropy, which is often less than $50 \text{ J K}^{-1} \text{ kg}^{-1}$. The Hadley circulation is characterized by a low-level equatorward and a high-level outflow. Because of the low effective stratification for moist entropy, the atmospheric circulation is very inefficient at exporting moist entropy or energy out of the equatorial regions, and, as argued by Czaja and Marshall (2006) most of the energy transport out of the equatorial regions must take place in the ocean.

In midlatitudes, however, the two stratifications are very close to each other. The argument for a lower moist stratification is based on the vertical variations of moisture (9) and assumes implicitly that the circulation is composed of an upper-level flow in one direction and a return flow in the opposite direction at a lower level. However, the moist stratification can be higher than the dry stratification if a flow of moist air is balanced by a return flow of drier air at the same temperature and pressure. In this case, the transport of moist entropy can be much larger than the transport of dry entropy. The effective stratifications in midlatitudes correspond to neither a direct circulation with upper-level poleward flow and low-level return flow (which would imply a low moist stratification) nor to a turbulent transport by horizontal eddies (which would imply a larger moist stratification). Rather, that the two effective stratifications are

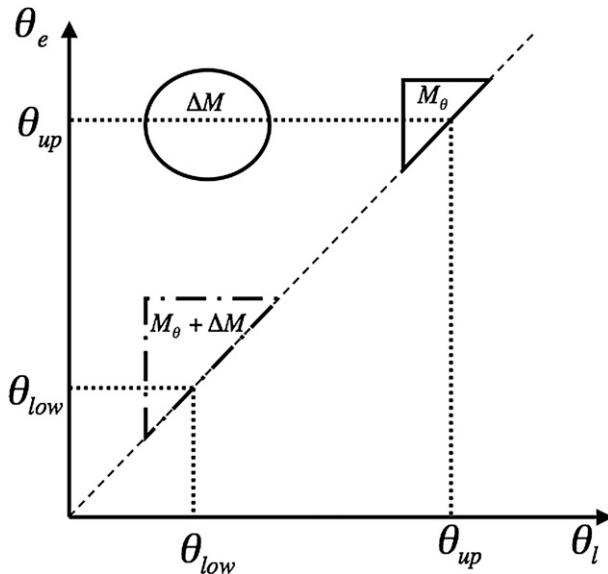


FIG. 8. Schematic representation of the distribution of the meridional mass transport on isentropic filaments in midlatitudes. See text for details.

comparable indicates that midlatitude flow is a mixture of direct and eddy-dominated circulations.

To explain why the effective stratifications for moist entropy and dry entropy are almost equal in midlatitudes, we consider an idealized flow described in Fig. 8, which shows an idealized distribution of the meridional mass transport on isentropic filaments, similar to Figs. 3 and 4. We assume that the total circulation on dry isentropes is $\Delta\Psi_{\theta_l} = M_\theta$. On dry isentropes, the poleward flux takes place in the upper troposphere at $\theta_l = \theta_{up}$ (solid line), and the return flow occurs in the lower troposphere at $\theta_l = \theta_{low}$ (dash-dot line). The additional mass transport on moist isentropes $\Delta M = \Delta\Psi_{\theta_e} - \Delta\Psi_{\theta_l}$ (solid circle) must take place in the lower troposphere on a dry isentrope with $\theta_l \sim \theta_{low}$, with the additional return flow taking place on the same isentrope. In the lower troposphere, the θ_e of a parcel is always larger than its θ_l . It must also be smaller than θ in the upper troposphere:

$$\theta_{low} \leq \theta_e \leq \theta_{up}.$$

Indeed, if this were not the case, the parcel would be convectively unstable. This means that in Fig. 8 the entire circulation must take place within the triangle bounded by the diagonal $\theta_l = \theta_e$, the vertical line $\theta_l = \theta_{low}$ and the horizontal line $\theta_e = \theta_{up}$.

In this schematic picture, the dry stratification is given by $\theta_{up} - \theta_{low}$. For the moist stratification to be equal to the dry stratification, the equivalent potential in the low-level poleward flow must be comparable to the upper-tropospheric potential temperature; that is, $\theta_e \approx \theta_{up}$. In

addition, the equivalent potential temperature in the return flow must be close to θ in the lower troposphere (i.e., $\theta_l \approx \theta_{low}$), which implies that the return flow has a low humidity content. Hence, most of the equatorward flux M_θ takes place near the surface, at low θ_l and θ_e : $\theta_l \approx \theta_e \approx \theta_{low}$, while the poleward flow is split between an upper branch with $\theta_e \approx \theta_l \approx \theta_{up}$ and a lower branch with $\theta_l \approx \theta_{low}$ and $\theta_e \approx \theta_{up}$. The fact that the dry and moist stratification are almost equal in midlatitudes is tied to the fact that the near-surface poleward flow is composed of air parcels with high θ_e that rise into the upper troposphere within the storm tracks.

5. Conclusions

By analyzing the circulation on isentropic surfaces, one can assess the trajectories of air parcels. A central issue discussed in this paper is that isentropic surfaces are not uniquely defined because of the variations in water content. Here, the circulation is analyzed on two sets of isentropic surfaces based on two distinct definitions for entropy: the dry entropy, which is closely related to the liquid water potential temperature, and the moist entropy, related to the equivalent potential temperature. While qualitatively similar, the circulations on dry and moist isentropes differ quantitatively. In the midlatitudes, the total mass transport on moist isentropes is significantly larger than the transport on dry isentropes. In the tropics, the mass transport on dry isentropes is larger than that on moist isentropes.

Pauluis et al. (2008) argue that in midlatitudes the additional mass flux on moist isentropic surfaces corresponds to a near-surface poleward flow of warm moist air with high θ_e and low θ . It is balanced by an equatorward return flow of dry air at low θ_e and low θ_l . The poleward mass transport in this low-level branch of the circulation is comparable to the net poleward mass transport in the upper troposphere. This surface branch also accounts for approximately half of the total energy and entropy transport by the atmospheric circulation.

Based on these findings, Pauluis et al. (2008) have proposed a revised version of the global circulation illustrated in Fig. 9. The circulation is subdivided into two main components. First, a global overturning cell similar to the dry isentropic circulation obtained by Held and Schneider (1999) is characterized by air rising in the equatorial regions, moving poleward in the upper troposphere, with some air subsiding within the subtropics, and the rest being transported to higher latitudes by the midlatitudes eddies. The air then subsides in the polar regions before returning toward the equator near the earth's surface. The second moist branch of the circulation starts with low-level warm moist air parcels being

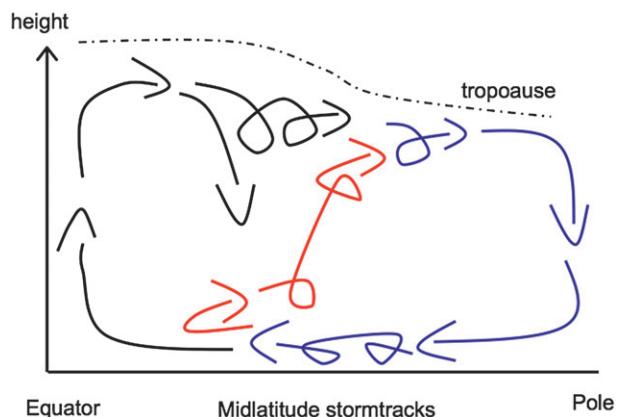


FIG. 9. Schematic representation of the global atmospheric circulation after Pauluis et al. (2008). Black and blue arrows indicate isentropic circulation on dry isentropes, with air rising at the equator, moving poleward in the upper troposphere, and returning equatorward near the surface. Red and blue arrows indicate the moist branch of the circulation corresponding to the additional mass transport on moist isentropes, with warm moist air moving toward the midlatitudes at low level, rising into the upper troposphere within the storm tracks, and subsiding over the pole, and dry cold air moving toward the equator near the surface.

advected from the subtropics into the storm tracks by the midlatitude eddies. These air parcels ascend into the upper troposphere within the storm tracks, where they then merge with the poleward flow at high levels. The rest of the circulation is then similar to the dry branch, with air subsiding over the polar regions and returning toward the equator near the earth's surface. This moist branch is conceptually similar to the Palmén–Newton circulation (Palmén 1951; Palmén and Newton 1969), which emphasizes the role of ascending air within the midlatitudes storm tracks.

This paper expands on the overall results of Pauluis et al. (2008) and further compares the global circulation on dry and moist isentropes. The general characteristics of the isentropic circulations, namely that the dry circulation is larger in the tropics and that the moist circulation is larger in the midlatitudes, are observed all year long and in both hemispheres. The seasonal cycle is present in both circulations. This is much more obvious in the dry circulation, which clearly shows the seasonal reversal of the cross-equatorial Hadley circulation from the summer hemisphere to the winter hemisphere. The midlatitudes of the Northern Hemisphere also exhibit a large seasonality: the circulation on dry isentropes is very strong during the winter but almost disappears during the summer. This summer weakening is much less pronounced in the circulation on moist isentropes, and as a result the mass transport on moist isentropes is approximately 3 times larger than the mass transport on dry isentropes in the Northern Hemisphere summer. In contrast there

is much less seasonal variation in midlatitudes of the Southern Hemisphere, a difference likely tied to the land–sea contrast between the two hemispheres.

The difference between the tropics, where the circulation on dry isentropes is larger, and the extratropics, where the circulation on moist isentropes is larger, can be attributed to the direction of the transport of warm moist air parcels near the surface. In midlatitudes, warm moist air parcels near the surface with high values of θ_e but low values of θ_i move on average toward the pole, in the same direction as the upper-tropospheric flows at high θ_i . But in the tropics, warm moist air parcels with high θ_e and low θ_i move on average toward the equator, in the opposite direction to the flow in the upper troposphere. The transition between a circulation with larger mass transport on dry isentropes and one with larger mass transport on moist isentropes occurs between 15° and 20° latitude. This transition can be interpreted roughly as the point at which eddies (including stationary eddies) begin to transport a significant amount of water vapor out of the subtropical regions.

The isentropic circulation is closely tied to an equator-to-pole entropy transport necessary to balance the differential heating of the earth's atmosphere. The two entropy transports are not equal, with circulations transporting more dry entropy in the equatorial regions and more moist entropy in the midlatitudes. The difference between the two entropy transports is proportional to the transports of water vapor by the circulation. This situation is analogous to the relationship between the transports of dry static energy, moist static energy, and latent heat. We also introduce an effective stratification as the ratio between the total entropy transport and the total mass transport in isentropic coordinates. Remarkably, the effective stratifications for moist entropy and dry entropy are approximately equal in midlatitudes. This finding is in apparent contradiction with the fact that the vertical variations of θ_e are always smaller than the vertical variation of θ . For the moist stratification to be comparable to the dry stratification, horizontal fluctuations of θ_e must be comparable to the vertical fluctuations of θ . This requires θ_e in the moist branch of the circulation to be characteristic of air parcels that are almost convectively unstable and ready to ascend to the upper troposphere.

The evidence for moist ascent in the midlatitudes that can be derived solely on the basis of the isentropic circulation is primarily indirect. Our arguments here rely on three key elements: 1) the large poleward mass transport of warm moist air in the subtropics, which should saturate as air parcels move both poleward and upward; 2) that the equivalent potential temperature in this low level is close to the potential temperature in the

upper troposphere, which indicates that the air parcels are close to be convectively unstable; and 3) that this low-level flow is associated with the poleward transport of latent heat, which is released through condensation and precipitation within the storm tracks. There has been plenty of evidence of rising saturated air parcels in mid-latitude eddies. The midlatitude ascent is consistent with rising motion in the warm conveyor belt of extratropical cyclones (Carlson 1980; Browning 1999; Stohl 2001) and with slantwise convective adjustment in midlatitude storms (Emanuel 1988). This is also consistent with the arguments proposed by Juckes (2000) and Frierson et al. (2006) that convective adjustment plays a direct role in establishing the midlatitude stratification (Frierson 2006, 2008). Korty and Schneider (2007) have diagnosed the climatology of air masses with lapse rates nearly neutral to moist convection, identifying those locations and times where air masses are neutral to gravitational convection or slantwise convection. They found such a state was common during winter in the ocean storm tracks. At this point, it is not possible to determine whether the midlatitude ascent occurs through rapid convective events or slower slantwise motions in baroclinic eddies. To resolve this issue, one would need much higher-resolution data, which resolve at least the frontal structure and possibly the convective scales. While more work must still be done, our analysis unambiguously confirms that ascent of warm moist air plays a major role in the dynamics of the midlatitude storm tracks.

The circulations on dry and moist isentropes differ significantly, though neither should be viewed as better than the other. Rather, as shown throughout this paper, new insights can be gained by contrasting them. By comparing the isentropic circulations obtained from numerical models and observations, one should be able to assess the model's ability to simulate some of the key features of the circulation. For example, differences in the dynamics of the storm tracks could be identified based on differences in the isentropic mass transports or stratifications. Combined together, the dry and moist isentropic circulations offer powerful diagnostic tools to analyze atmospheric motions. Conversely, in a coupled ocean–atmosphere model, analysis of the moist atmospheric circulation and its oceanic counterpart can reveal differences as to which fluid dominates the poleward energy transport (e.g., Czaja and Marshall 2006). Overall, we anticipate that simplified climate models can be built from this new way of looking at atmospheric motions.

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APPENDIX A

Entropy of Moist Air

In atmospheric sciences, cloudy air is usually treated as a mixture of dry air, water vapor, and condensed water (either liquid water or ice). The specific humidity q_v , the specific humidity for condensate water q_l , and the specific humidity for total water q_T are, respectively, the mass of water vapor, condensed water, and total water per unit mass of cloudy air. The mass of dry air per unit mass of cloudy air is thus $1 - q_T$. The specific entropy of moist air S is equal to the weighted average of the specific entropies of dry air s_d , water vapor s_v , and condensed water s_l :

$$S = (1 - q_T)s_d + q_v s_v + q_l s_l. \quad (\text{A1})$$

The specific entropies of the individual components are

$$s_d = C_{pd} \ln \frac{T}{T_0} - R_d \ln \frac{p_d}{p_0} + s_{d,0}, \quad (\text{A2a})$$

$$s_v = C_{pv} \ln \frac{T}{T_0} - R_v \ln \frac{e}{e_0} + s_{v,0}, \quad (\text{A2b})$$

$$s_l = C_l \ln \frac{T}{T_0} + s_{l,0}. \quad (\text{A2c})$$

Here, C_{pd} , C_{pv} , and C_l are the specific heat capacities at constant pressure of dry air, water vapor, and liquid water; R_d and R_v are the ideal gas constants for dry air and water vapor; T is the air temperature; and p_d and e are the partial pressure of dry air and water vapor. These expressions are obtained by assuming that both dry air and water vapor behave as an ideal gas and by neglecting the specific volume of condensed water. The definitions of the specific entropies [(A2a)–(A2c)] include several integration constants: the reference temperature T_0 , reference partial pressure for dry air p_0 , and water vapor e_0 , and the reference entropies for dry air $s_{d,0}$, water vapor $s_{v,0}$, and condensed water $s_{l,0}$. The expressions for the specific entropies here are such that they are equal to their reference values when the component is at the reference temperature and partial pressure. A common practice is to choose some typical atmospheric values for T_0 and partial pressures $p_{d,0}$ and e_0 , and to set the reference value for the specific entropies of dry air and of either liquid water or water vapor to 0.

The integration constant for water vapor and liquid water cannot be simultaneously set to 0 because the entropy difference between liquid water and water vapor at saturation must be

$$s_v - s_l = \frac{L_v}{T}.$$

From a practical point of view, it is useful to choose either the specific entropy of water vapor or of liquid water to be 0 in the reference state. The former can be achieved by setting the integration constant $s_{c,0}$ to zero (as well as $s_{d,0}$). This leads to the standard definition of the moist entropy S_m :

$$S_m = [(1 - q_T)C_{pd} + q_T C_l] \ln \frac{T}{T_0} - (1 - q_T)R_d \ln \frac{p_d}{p_0} + q_v \frac{L_v}{T} - q_v R_v \ln \frac{e}{e_s(T_0)}. \quad (\text{A3})$$

An alternative is to set the integration constant $s_{v,0}$ to 0 so that the entropy of cloudy air is given by the dry entropy S_l :

$$S_l = [(1 - q_T)C_{pd} + q_T C_{pv}] \ln \frac{T}{T_0} - (1 - q_T)R_d \ln \frac{p_d}{p_0} - q_v R_v \ln \frac{e}{e_0} + q_l \frac{L_v}{T} - q_l R_v \ln \frac{e_s(T)}{e_0}. \quad (\text{A4})$$

These expressions for the dry entropy and moist entropy differ solely in the choice of the reference state for the water. In particular, the difference between these two entropies is

$$S_m - S_l = q_T \left[\frac{L_v(T_0)}{T_0} - R_v \ln \frac{e_0}{e_s(T_0)} \right] = q_T \delta s_0. \quad (\text{A5})$$

It should be noted that the entropy used in atmospheric sciences corresponds to the thermodynamic entropy in classical physics. It is a state variable that can be used in Clausius formulation of the second law. It does not, however, correspond to the absolute entropy based on Nernst's theorem, which states that the entropy of a system should go to zero as the absolute temperature goes to zero. The expression for the entropy of an ideal gas in classical physics $S = C_p \ln T - r \ln p + S_0$ is fundamentally incompatible with Nernst theorem as it is singular as $T \rightarrow 0$. This results from the fact that the classic concept of an ideal gas neglects quantum effects that are important at low energy and temperature.¹

¹ Nernst's theorem is tied to the fact that, in quantum physics, there is a finite energy difference between individual quantum states. At very low temperature, thermal fluctuations are too weak to induce any state transition. It is not clear whether Nernst's theorem should apply to systems in which energy levels form a continuum as is the case for an ideal gas in classical physics.

APPENDIX B

Entropy and the Second Law of Thermodynamics

The second law of thermodynamics requires that the entropy of an air parcel be conserved for closed, reversible adiabatic transformations. In addition, for closed processes in presence of either energy exchange or irreversibility, the second law of thermodynamics can be written as

$$\rho \frac{dS}{dt} = \frac{Q}{T} + \dot{S}_{\text{irr}}, \quad (\text{B1})$$

where Q is the external heating rate per unit volume, and \dot{S}_{irr} is the irreversible entropy production by atmospheric processes, which is always positive. Both the dry entropy S_l or the moist entropy S_m are valid definitions of entropy and can be used in formulating the second law of thermodynamics.

In the atmosphere, however, many processes including precipitation, evaporation, and diffusion involve exchange of water between air parcels or with the surface. An open transformation is a process in which water in either phase is added or removed from a parcel. If \dot{q}_v is the rate of change of specific humidity owing to diffusion and \dot{q}_l the rate of change of the specific humidity for condensed water owing to precipitation, the entropy tendency becomes

$$\rho \frac{dS}{dt} = \frac{Q}{T} + \dot{S}_{\text{irr}} + (s_v - s_d)\rho\dot{q}_v + (s_l - s_d)\rho\dot{q}_l. \quad (\text{B2})$$

The tendencies \dot{q}_v and \dot{q}_c do not include phase transitions per se, as these correspond to internal transformations and are already accounted for in (B1). Equation (B2) applies to both the dry entropy S_l and moist entropy S_m and differs only by the value for the entropy of condensed water s_l and water vapor s_v . Because moist entropy and dry entropy rely on different reference values for the specific entropies of water vapor and condensed water, they are also affected differently by evaporation and precipitation.

In the definition of moist entropy, the reference state is chosen such that the specific entropy of the condensed water is small, but the entropy of water vapor is much larger because of the term L_v/T . In the entropy tendency equation, the term proportional to L_v/T is much larger than any of the other ones in $s_v - s_d$ and in $s_l - s_v$. Hence, if we keep only the terms proportional to the latent heat in the entropy tendency, we get

$$\rho \frac{dS_m}{dt} \approx \frac{Q}{T} + \dot{S}_{\text{irr}} + \frac{L_v}{T} \rho \dot{q}_v. \quad (\text{B3})$$

Evaporation acts as a source of moist entropy that is approximately proportional to the surface latent heat flux. By contrast, precipitation has little effect on moist entropy.

In the case of the dry entropy, the reference state for water is such that the entropy of water vapor is close to that of dry air, while that of liquid water is much smaller owing to the latent heat contribution $-L_v/T$. If we retain only the contribution from the terms proportional to the latent heat in $s_v - s_d$ and $s_l - s_d$, we can approximate the dry entropy tendency by

$$\rho \frac{dS_l}{dt} \approx \frac{Q}{T} + \dot{S}_{\text{irr}} - \frac{L_v}{T} \rho \dot{q}_c. \quad (\text{B4})$$

While evaporation has little effect on the dry entropy, precipitation acts as a large source of the dry entropy. The entropy increase is approximately proportional to the net latent heat released by the condensation. (It should be stressed, however, that the entropy increase occurs when condensed water is removed, and not when water vapor condenses.) Thus, in contrast to the moist entropy, the dry entropy is not conserved during precipitating convection.

APPENDIX C

Alternative Choice for the Entropy and the Global Circulation

Any choices for the integration constants in (A2a)–(A2c) yield a valid definition of the entropy of moist air, say S_a , that differs from either S_m and S_l by a constant multiplied by q_T . Because of (A5), it can be expressed as a linear combination of S_m and S_l :

$$S_a = (1 - a)S_m + aS_l,$$

where a is an arbitrary constant. The streamfunction on surfaces of constant value of S_a would be different from the streamfunctions on dry and moist isentropes. This raises an important question: what aspects of the isentropic circulation do not depend on the integration constant?

Our answer is in that, rather than isentropic surfaces, one should think in terms of isentropic filaments defined as regions where both entropies S_m and S_l are constant. As S_m and S_l are constant along such isentropic filaments, so are all possible definitions of the entropy of moist air. If isentropic filaments are defined using different definitions of entropy—for example $S_a = (1 - a)S_m + aS_l$ and $S_b = (1 - b)S_m + bS_l$, with $a \neq b$ —the meridional mass transport on an isentropic filament defined by (S_a, S_b) is given by

$$\begin{aligned} M_{\text{ab}}(S_a, S_b, \phi) dS_a dS_b &= M_{\text{ml}}(S_m, S_l, \phi) dS_m dS_l \\ &\rightarrow M_{\text{ab}}(S_a, S_b, \phi) \\ &= |a - b| M_{\text{ml}}(S_m, S_l, \phi). \end{aligned}$$

Unlike streamfunction, the distribution of the meridional mass transport on isentropic filaments does not depend on any specific definitions for the entropy of moist air.

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