

A box Climate Model: *EPcm*

Environmental Physics 2009-2010

Model documentation –v4

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http://www.sp.ph.ic.ac.uk/~arnaud/EP_ClimateModel.html

0. Motivation

The following is a simple Two-Box Climate model, designed for pedagogical purposes. It aims at predicting the time dependent response of Tropical and Extra-Tropical surface temperatures to a given time-dependent change in atmospheric greenhouse gas concentration. The model includes a representation of atmospheric and oceanic heat transport and storage, and a single positive feedback: the water vapour feedback. The model is coded with MATLAB (see http://www.sp.ph.ic.ac.uk/~arnaud/EP_ClimateModel.html for download)

References

- Emanuel, K., 2002: A simple model of multiple climate regimes, *J. Geophys. Res.*, vol 107.
- Czaja A., and J. C. Marshall, 2006: The partitioning of heat transport between the ocean and atmosphere, *J. Atm. Sci.*, vol 63, 1498-1511.
- Held, I. and B. J. Soden, 2006: Robust response of the hydrological cycle to global warming, *J. Clim.*, vol 19, 5686-5699.

Version 3 improvements

- Improved representation of oceanic heat transport and storage (separate evolution equations for surface and thermocline temperature)
- Improved representation of atmospheric heat transport with a simple diagnostic hydrological cycle.
- Simple representation of climate “noise” (stochastic component added to the surface turbulent heat flux)

1. Model variables

Average atmospheric temperature (whole column of air)

$$T_a \equiv \frac{1}{P_s} \int_0^{P_s} T dP \quad \text{in which } P_s \approx 10^5 Pa \text{ is surface pressure.}$$

Surface temperature (assumed to be ocean)

$$T_s \equiv \frac{1}{h_m} \int_{-h_m}^0 T dz \quad \text{in which } h_m \approx 50m \text{ is the ocean surface (or "mixed") layer.}$$

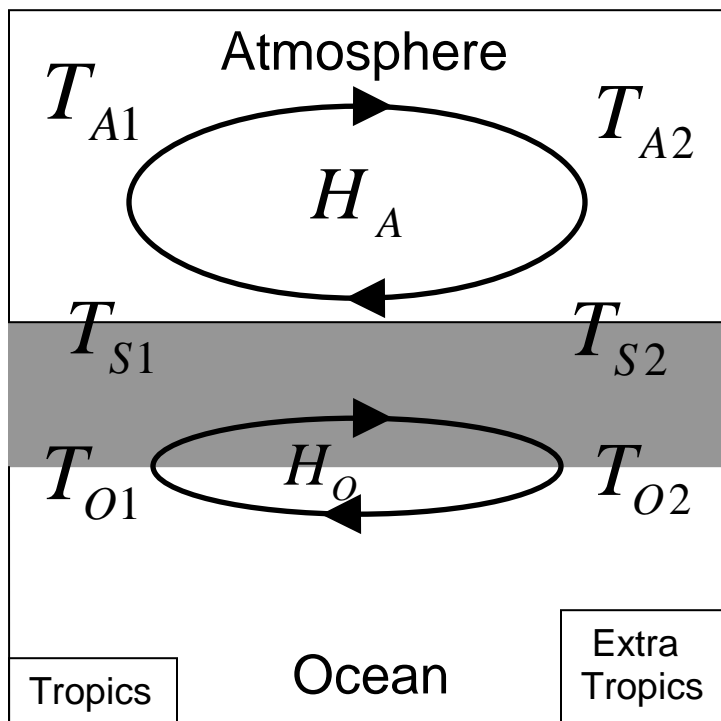
Average oceanic temperature (below the mixed layer)

$$T_o \equiv \frac{1}{h_o} \int_{-(h_o+h_m)}^{-h_m} T dz \quad \text{in which } h_o \approx 500m \text{ is the warm layer ("thermocline") thickness.}$$

Atmospheric and oceanic heat transport

$$H_A = C_A \psi_A \Delta T_A \quad \& \quad H_A = C_o \psi_o \Delta T_o$$

in which $C_{A,o}$ is a heat capacity, $\psi_{A,o}$ measures the strength of the circulation and ΔT is the temperature difference between Tropics (Eq-30N) and Extra-Tropics (30N-90N). Note that Ψ_o refers to the circulation of waters between the mixed layer and the thermocline (wind driven circulation). No thermohaline effect is included in the model.



Model geometry

A tropical (Box 1) and an extra-tropical (Box 2) region are considered. The oceanic region is further decomposed into a surface and a deep (thermocline) layer. There are no lands and all surface fluxes represent air-sea exchanges. The model is solely driven by solar energy input and atmospheric CO2 concentration (water vapour is interactive). Only one hemisphere is considered.

2. Model Equations

Top of the atmosphere radiative fluxes (Box 1&2)

$$F_{T1} = \sigma T_{E1}^4 - (\varepsilon_1 \sigma T_{A1}^4 + (1 - \varepsilon_1) \sigma T_{S1}^4) \quad [F_T] = Wm^{-2}$$

$$F_{T2} = \sigma T_{E2}^4 - (\varepsilon_2 \sigma T_{A2}^4 + (1 - \varepsilon_2) \sigma T_{S2}^4)$$

$$\varepsilon_1 = 1 - e^{-(\alpha CO_2 + \gamma q_1)} \quad \text{with } q_1 = 1000 \times RH_1 q_{sat} \left(\frac{T_{S1} + T_{A1}}{2}, 750mb \right)$$

$$\varepsilon_2 = 1 - e^{-(\alpha CO_2 + \gamma q_2)} \quad \text{with } q_2 = 1000 \times RH_2 q_{sat} \left(\frac{T_{S2} + T_{A1}}{2}, 750mb \right)$$

Turbulent surface fluxes (vertical convection parameterization=sensible+latent)

$$F_{TS1} = \Lambda (T_{S1} - T_{A1} - \Delta T_Z) \quad \text{if } (T_{S1} - T_{A1} - \Delta T_Z) > 0, \quad F_{TS1} = 0 \quad \text{otherwise} \quad [F_{TS}] = Wm^{-2}$$

$$F_{TS2} = \Lambda (T_{S2} - T_{A2} - \Delta T_Z) \quad \text{if } (T_{S2} - T_{A2} - \Delta T_Z) > 0, \quad F_{TS2} = 0 \quad \text{otherwise}$$

NB: The parameter Λ is chosen as a random variable. This allows a simple representation of "noise". The net surface heat flux F_S the sum of the turbulent and radiative fluxes.

Atmospheric circulation strength (diffusive parameterization)

$$\psi_A = K_A (T_{S1} - T_{S2}) \quad [\psi_A] = kgs^{-1}$$

Heat transports (Ocean & Atmosphere)

$$H_A = \psi_A (h_{A1} - h_{A2}) \quad \text{in which } h_A = c_p T_A + l_v q_a \quad \text{is moist static energy at low level} \quad [H_A] = W$$

NB: The neglect of gravitational potential is consistent with quasi-geostrophic baroclinic waves heat transport

$$F_A = H_A / \pi R^2 \quad [F_A] = Wm^{-2}$$

$$H_O = C_O \psi_O (T_{S1} - T_{O2}) \quad [H_O] = W$$

Energy conservation for Atmosphere (Box 1&2)

$$C_A \frac{P_S}{g} \frac{dT_{A1}}{dt} = (F_{T1} + F_{S1}) - F_A$$

$$C_A \frac{P_S}{g} \frac{dT_{A2}}{dt} = (F_{T2} + F_{S2}) + F_A$$

NB: A simple parameterization of moist static energy is used here ($C_A \approx 2c_{pA}$).

Energy conservation for Ocean (Box 1&2)

$$\rho_O C_O h_m \frac{dT_{S1}}{dt} = -F_{S1} + \Psi_O c_o (T_{O1} - T_{S1}) / \pi R^2 \quad \rho_O C_O h_o \frac{dT_{O1}}{dt} = \Psi_O c_o (T_{O2} - T_{O1}) / \pi R^2$$

$$\rho_O C_O h_m \frac{dT_{S2}}{dt} = -F_{S2} + \Psi_O c_o (T_{S1} - T_{S2}) / \pi R^2 \quad \rho_O C_O h_o \frac{dT_{O2}}{dt} = \Psi_O c_o (T_{S2} - T_{O2}) / \pi R^2$$

3. Model diagnostics

Hydrological cycle

The poleward transport of moisture is computed as

$$F = \Psi_A (q_{A1} - q_{A2})$$

while the surface evaporation is given by the convective parameterization above. Thus, under the assumption of steady state moisture budget, one can deduce the precipitation in each box as a residual,

$$F = E_{1,2} - P_{1,2}$$

The fixed relative humidity assumption implies that the Clausius-Clapeyron scaling is strictly obeyed, but this does not have to be the case for precipitation.

The global evaporation $E = (F_{S1} + F_{S2}) / 2$ (equal to the global precipitation) is taken as a measure of the hydrological cycle.

NB: All specific humidity calculations are carried out at a prescribed low level pressure of 750mb with imposed relative humidity of 0.6.

Vertical ocean heat transport

Summing the equation for the two ocean surface layers we get an evolution equation for the heat content of the upper ocean,

$$\frac{d}{dt} [\rho_o c_o h_m (T_{S1} + T_{S2})] = -(F_{S1} + F_{S2}) + \frac{\Psi_o c_o}{\pi R^2} (T_{O1} - T_{S2})$$

Conversely for the heat content of the deep ocean,

$$\frac{d}{dt} [\rho_o c_o h_o (T_{O1} + T_{O2})] = -\frac{\Psi_o c_o}{\pi R^2} (T_{O1} - T_{S2})$$

The term $Q_o \equiv \Psi_o c_o (T_{O1} - T_{S2}) / \pi R^2$ thus measures the strength of the vertical exchange of heat between the upper and deep ocean (it does not change the total heat content, simply redistribute heat in the vertical). The associated sensitivity is $\lambda_o \equiv \Psi_o c_o / \pi R^2 \approx 0.3 \text{ W m}^{-2} \text{ K}^{-1}$ for the control simulation.

4. Model control parameters

Ocean:

Thickness of mixed layer

$$h_m = 50m$$

Thickness of thermocline layer

$$h_o = 500m$$

Atmosphere:

Low level relative humidity (Box 1&2)

$$RH_1 = RH_2 = 0.6$$

Effective heat capacity

$$C_A = 2000 Jkg^{-1}K^{-1}$$

Critical vertical temperature gradient

$$\Delta T_Z = 40K$$

Circulation strength parameter

$$K_A = 100/15 SvK^{-1}$$

Emissivity parameter for water vapour

$$\gamma = 1.25$$

Emissivity parameter for carbon dioxide

$$\alpha = 1.2 \times 10^{-3}$$

Carbon concentration

$$CO_2 = 280 ppm$$

Ocean / Atmosphere coupling:

Ratio of mass transport

$$\psi_O / \psi_A = 0.1$$

Solar input:

Emission temperature (Box 1)

$$T_{E1} = 268K$$

Emission temperature (Box 2)

$$T_{E2} = 240K$$

Fudge factor

Large number for convective parameterizations

$$\Lambda = 100 \times [1 + 0.05\xi(t)]$$

5. Model simulations

(a) Control Climate results

Numbers:

Surface temperature of Tropics	$T_{S1} = 299.89K$
Surface temperature of Extra-Tropics	$T_{S2} = 280.69K$
Global surface temperature	$T_S = 290.29K$
Low level moisture of Tropics	$q_1 = 4.78g / kg$
Low level moisture of Extra-Tropics	$q_2 = 1.07g / kg$
Thermocline temperature (Tropics)	$T_{O1} = T_{S2}$
Thermocline temperature (Extra-Tropics)	$T_{O2} = T_{O1} = T_{S2}$
Atmospheric circulation strength	$\psi_A = 128 \times 10^9 kg s^{-1}$
Atmospheric moisture transport	$F = 0.47 \times 10^9 kg s^{-1}$
Atmospheric heat transport	$H_A = 3.54PW$
Oceanic heat transport	$H_O = 0.98PW$
Total Heat transport	$H_A + H_O = 4.52PW$
Hydrological cycle	$E = 40 \pm 2.3Wm^{-2}$

Comments:

(i) The mean climate produces realistic numbers but the natural variability is much weaker than in the real world (especially for the surface temperature). This is because of the very “static” dynamics used to parameterize the oceanic and atmospheric circulations.

(ii) In steady state, energy conservation requires that $T_{S2} = T_{O2} = T_{O1}$. The model ocean is thus characterized by a pool of warm water (surface low latitudes) and a very large pool of cold water (high latitudes at the surface and the whole thermocline).

(iii) The mean net radiative flux at the TOA $F_T = (F_{T1} + F_{T2}) / 2$ is zero, in agreement with the fact that a steady state has been reached, but it fluctuates weakly on daily timescales with an rms of $0.03Wm^{-2}$.

(iv) The surface temperature is low enough at high latitudes that the atmospheric temperature profile is stable and convection occurs only very infrequently (i.e., $F_{S2} = 0$).

(b) Doubling CO2 Experiment minus Control

Numbers (change compared to control experiment, at equilibrium):

Surface temperature of Tropics	$\Delta T_{S1} = 1.57K$
Surface temperature of Extra-Tropics	$\Delta T_{S2} = 3.8K$
Global surface temperature	$\Delta T_S = 2.69K$
Atmospheric circulation strength	$\Delta \psi_A = -15 \times 10^9 \text{ kg s}^{-1}$ or $\Delta \psi_A / \psi_A = -4.3\%$ per K warming
Global low level moisture	$\Delta q / q = +6\%$ per K warming
Atmospheric moisture transport	$F = 0.44Sv$ or $\Delta F / F = -2.8\%$ per K warming
Atmospheric heat transport	$\Delta H_A = -0.56PW$ or $\Delta H_A / H_A = -5.9\%$ per K warming
Oceanic heat transport	$\Delta H_O = -0.21PW$ or $\Delta H_O / H_O = -8.1\%$ per K warming
Total Heat transport	$\Delta(H_A + H_O) = -0.77PW$
Hydrological cycle	$E = 42.9 \pm 2.27Wm^{-2}$ or $\Delta E / E = +2.68\%$ per K warming

Comments:

(i) EPcm climate sensitivity is on the order of 2.7K, in the range of most climate models.

(ii) The global moisture content scales roughly with Clausius-Clapeyron, as expected (for the global average surface temperature of the model, the latter is +6.4% per K warming).

(iii) The polar amplification of climate change is quite pronounced and is responsible for the reduction in the equator-to-pole temperature gradient. As a result, the strength of the atmospheric circulation weakens, driving similar trends in heat and moisture transports.

(iv) The surface high latitudes (Box 2) now start convecting (as opposed to the control experiment) as a result of being warmer and more unstable. This is the main reason why E increases (in the Tropics, surface evaporation decreases slightly, probably as a result of being a bit more stratified –reduction in atmospheric heat transport). The rate of increase is less than the Clausius-Clapeyron scaling, in agreement with climate model results (+2% per K warming).

(v) The dynamical oceanic timescale is on the order of a few centuries, very long for a wind – driven circulation (more realistically on the order of a decade or so). This reflects the waterworld geometry of the model (very big ocean basin!).

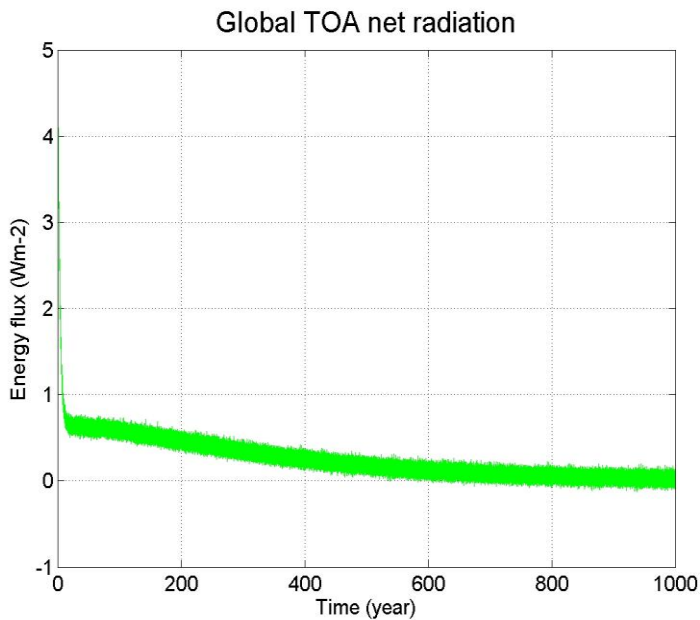


Fig. 1 Global net energy flux at the TOA. After a rapid initial decrease, it takes several centuries for the climate to warm up sufficiently that it can completely offset the initial trapping of longwave radiation induced by a CO₂ doubling. The excess energy is stored in the ocean –see Fig. 2. The internal variability is much weaker than the initial CO₂ forcing and so is only seen once the energetic imbalance has been reduced to less than 1 Wm^{-2} .

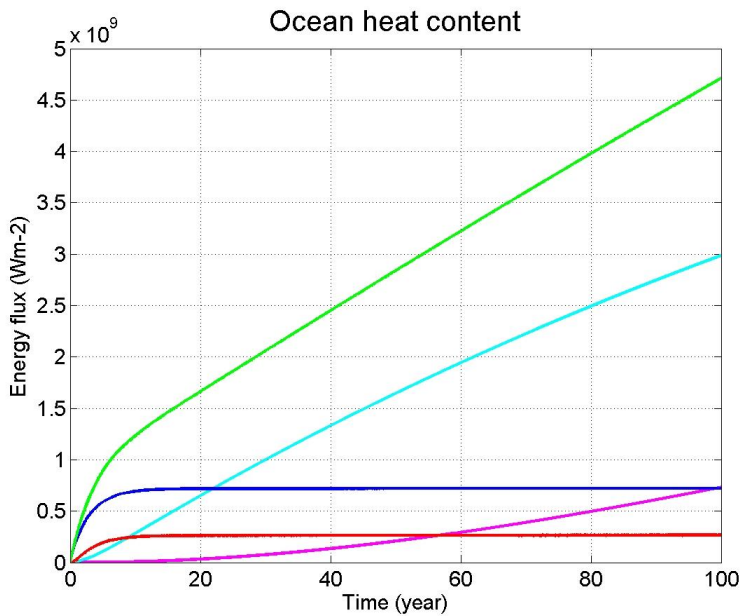


Fig. 2a First 100 years of ocean heat content timeseries (red=mixed layer Tropics; blue = mixed layer Extra-Tropics; magenta=thermocline Tropics; Cyan = thermocline Extra-Tropics; green = red+blue+magenta+cyan). It only takes a few years to establish the warmer mixed layer, but much longer to warm up the thermocline. Note from $t=0$ onwards there is no change in CO₂, even though a lot is still happening in the ocean. After about 20 years all the excess heat is stored in the the thermocline.

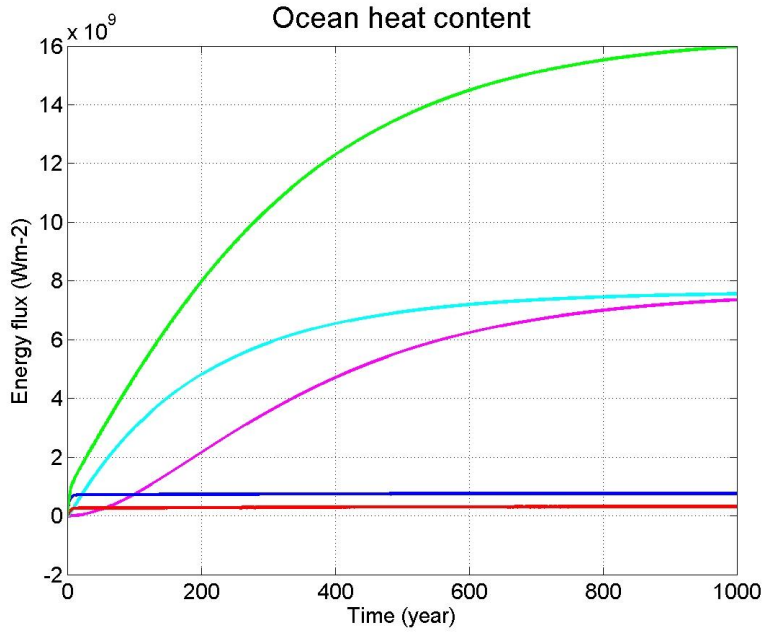


Fig. 2b Same as Fig. 2a but for the 1000-yr long timeseries. Of all curves the high latitude thermocline (cyan) is the one that shows the largest signal.

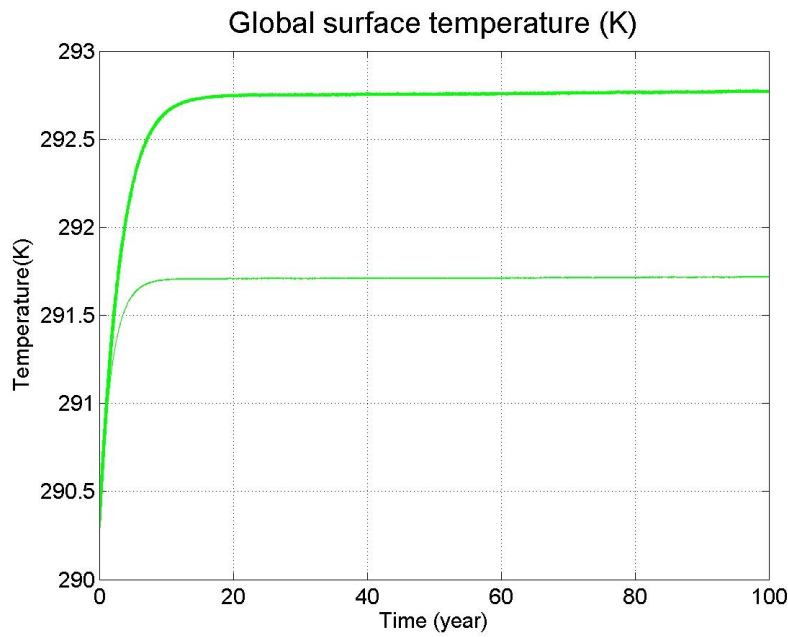
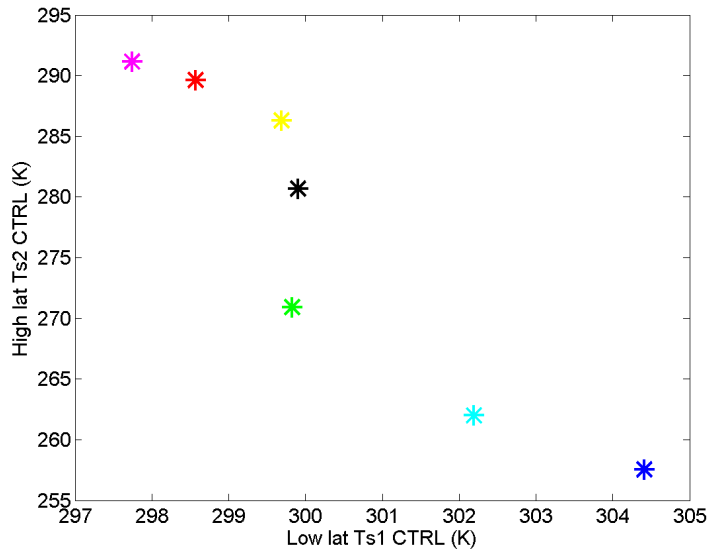
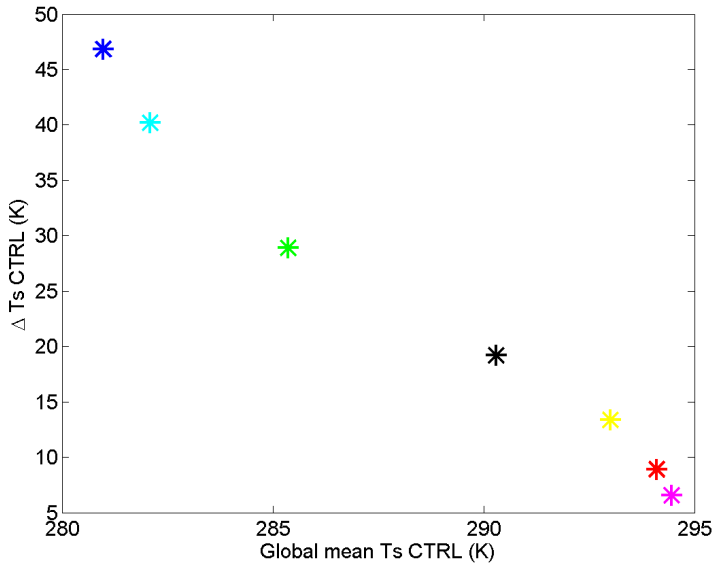


Fig. 3 Surface temperature change in response to a sudden doubling of atmospheric CO_2 concentration. The thick curve corresponds to the experiment described in Figs 1 and 2 while the thin curve is the temperature change in absence of water vapour feedback. The smaller change and the shorter timescale of adjustment are clearly visible.

(c) Sensitivity of the model mean state to the heat transport diffusivity K_A



Each star represents a model equilibrium state, but with a different value of K_A (control value multiplied by 1/10, 1/5, 1/2, 1 (black), 2, 5, 10 going from blue to magenta). The range of low latitude temperature change is only -6K while it is about +40K for the high latitudes. This reflects the weaker change in optical depth at low (already moist) compared to high latitudes resulting from increased transport of water vapour.



The weaker change in surface temperature at low compared to high latitudes implies a reduction in the equator to pole temperature difference as K_A increases. It also leads to a net increase in global average surface temperature (low latitudes cool less than high latitudes warm as K_A increases)

6. MATLAB code

Generalities

The Two-Box climate model is modular. The main program is `TwoBox_Main.m`. In it, you decide the length of integration and the model forcing (emission temperatures T_{E1}, T_{E2} ; CO2 concentration). The other important routine is `TwoBox_Param.m` in which you set the value of the model parameters. These are the two main routines you are likely to change when playing with the model. Some experiments are already stored on the website and can be used to initialize the model (MATLAB binary files):

- (i) `Output_CTRLv3.mat` (the model control simulation as described in this document)
- (ii) `Output_2XCO2FromCTRLv3_YR0_100.mat`: first 100 years of the 2XCO2 experiment (CO2 concentrations abruptly twiced at $t=0$).

Model Architecture:

<code>TwoBox_Main.m</code>	
<code>TwoBox_Param.m</code>	set model parameters
<code>TwoBox_Init.m</code>	initial conditions before time integration (default is CTRL run)
<code>TwoBox_Run.m</code>	time-loop over N_t timesteps
<code>calc_EPSA.m</code>	compute emissivities
<code>calc_Psi.m</code>	compute strength of atmospheric & oceanic circulations
<code>calc_Fs.m</code>	compute net surface heat flux
<code>calc_Ft.m</code>	compute net TOA radiative flux
<code>calc_Fa.m</code>	compute Atmospheric heat transport
<code>calc_Fo.m</code>	compute Oceanic heat transport
<code>TwoBox_Diagnos.m</code>	a few diagnostics (global average, etc)
<code>TwoBox_Plots.m</code>	displays model outputs
<code>TwoBox_Save.m</code>	store model outputs in MATLAB binary file

Running the model

Just decide the length of integration and the CO2 forcing in `TwoBox_Main.m` and then type

```
>> TwoBox_Main
```

on the MATLAB window. Enjoy!