# Instrumentation

Chris Carr

Monday 29th November 2010

# Programme for the PG Instrumentation Lectures

- Monday 29th November 10:00 - 11:30
  **Principles of Instrumentation (Chris)**

- Monday 29th November 14:00 - 15:30
  **Magnetic Field Instruments (Patrick)**

- Wednesday 8th December 10:00 - 12:00
  **Student presentations and Q&A Session (Juliet)**
  - For this you should prepare a 10-minute presentation about the instrumentation you are either
    a. Using for your project or
    b. Which generated the data you are using

# Your presentation should cover some or all of these questions, at least in outline:

1. How is the measurement made? What is the physical principle?
2. What is the range, resolution and bandwidth of the measurement?
3. How is the data calibrated, and what steps are taken to control both systematic and random errors?
4. What is the accuracy of the measurement?
5. What are the sources of noise? How is this minimised?
6. Will these instrumentation considerations impose limits on your work?

- *Your projects cover a wide range of disciplines and techniques, so not all of these questions are appropriate*
- *If you will be working with multiple data sets, you may like to concentrate on one measurement which is central to your work*
- *For guidance don’t hesitate to contact either Juliet or myself!*

# Some Principles of Instrumentation

This introductory lecture has four parts intended to help you answer these questions

1. A Fourier understanding of Signals and Instrumentation
2. Instrument characteristics and calibration
3. Sampled and digitised signals
4. Noise

# Preliminary Comments

- The relationship between the time and frequency domain is a central and recurring theme in instrumentation
- Therefore we will
  - revise Fourier theory and
  - Develop a Fourier understanding of signals and instruments

# Part 1 of 4
# Fourier Representation of Signals

- Assuming our signal $f$ is a function of time
- We must always consider the spectral content of our signal

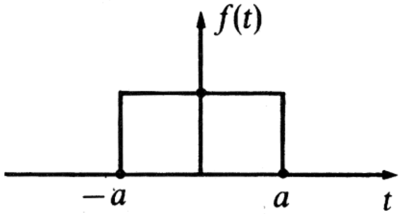
$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} dt$$

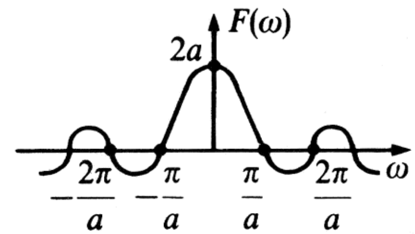
  - Note engineers use $j$ to avoid confusion with current $i$

# Pulse Function

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega \qquad F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$$



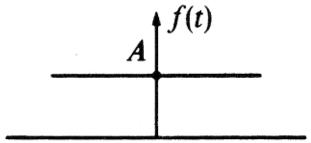

$$f(t) = \begin{cases} 1 & |t| \le a \\ 0 & \text{otherwise} \end{cases}$$



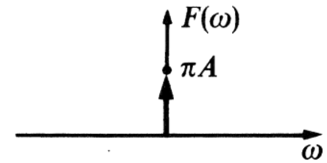

$$F(\omega) = 2\,\frac{\sin \omega a}{\omega}$$

- Finite in time
- Infinite in frequency

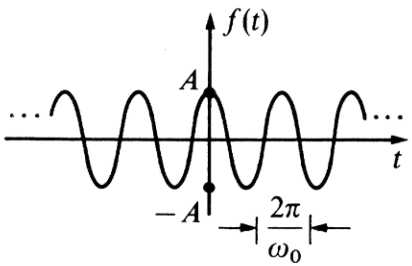
# Theoretical (infinite) Signals



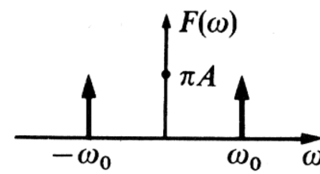

$f(t) = A$




$F(\omega) = 2\pi A\ \delta(\omega)$



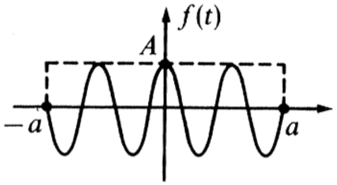

$f(t) = A \cos \omega_0 t$



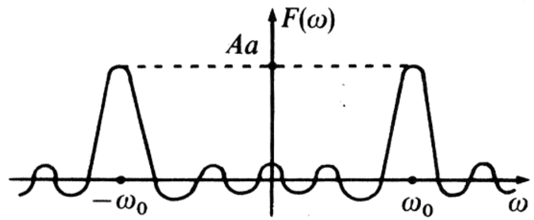

$F(\omega) = \pi A[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$

- Conclude:
  - There is no such thing as a DC measurement
  - Constant or Repetitive input has well defined spectrum
- Reality is or course more complex…

# Finite Sinusoid



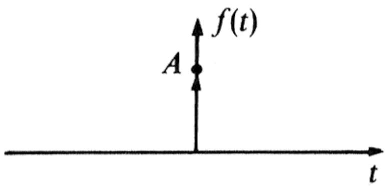

$$f(t) = \begin{cases} A \cos \omega_o t & |t| \leq a \\ 0 & \text{otherwise} \end{cases}$$



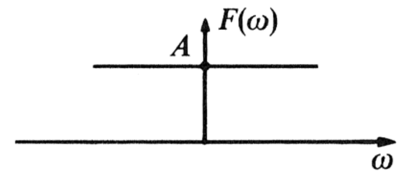

$$F(\omega) = A\left[\frac{\sin a(\omega - \omega_0)}{\omega - \omega_0} + \frac{\sin a(\omega + \omega_0)}{\omega + \omega_0}\right]$$

- End-effects
- In time-domain, the sinusoid is multiplied with a pulse
- In frequency-domain, the spectra are convolved

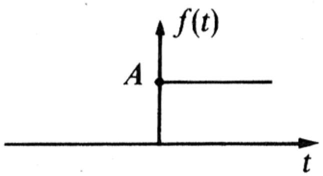
# The mathematical form of some real input signals used to test instruments



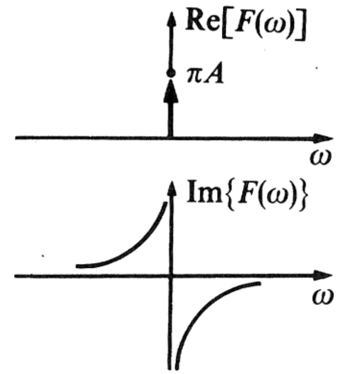

$$f(t) = A\ \delta(t)$$




$$F(\omega) = A$$



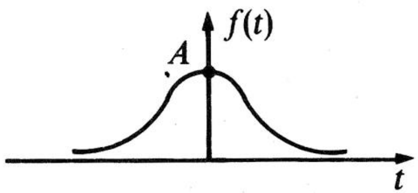

$$f(t) = \begin{cases} A & t > 0 \\ 0 & \text{otherwise} \end{cases}$$



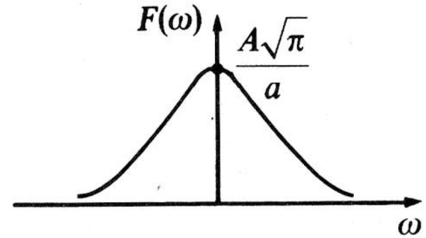

$$F(\omega) = A\left[\pi\ \delta(\omega) - j\,\frac{1}{\omega}\right]$$

- Discontinuity at $t = 0$
- Impulse: When applied to an instrument, **stimulates all frequencies simultaneously**
- Step input: more **physically realisable**

# Gaussian




$$f(t) = Ae^{-a^2t^2}$$

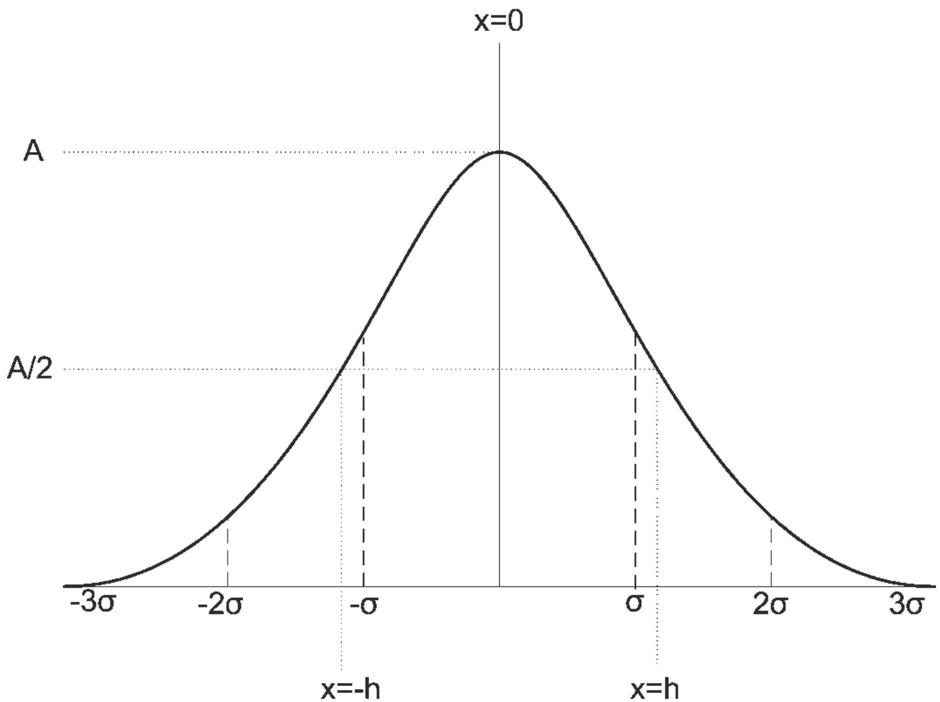



$$F(\omega) = A\,\frac{\sqrt{\pi}}{a}\,e^{-(\omega/2a)^2}$$

- Fourier Transform of a Gaussian is a Gaussian
- Infinite in time and frequency domains

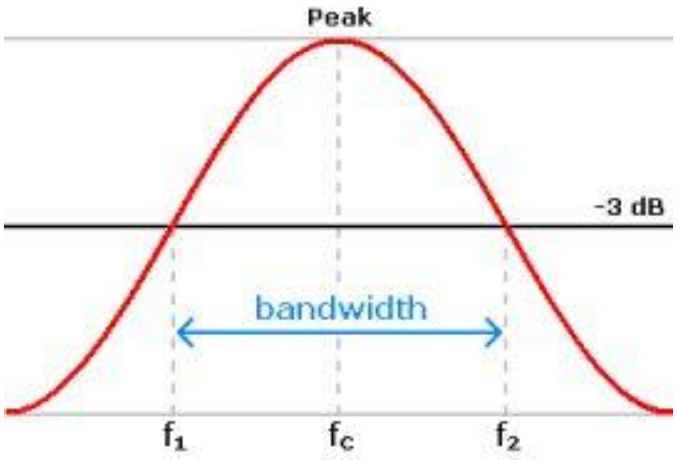
- Question: how ‘wide’ are these shapes?

# Width of an “Infinite” Pulse




- For signal **power** (or intensity)
  Use Full-Width Half-Maximum definition
  - E.g. Gaussian power-profile from a pulsed-laser
  - Duration is $2.35\sigma$

# Bandwidth is width in the frequency domain




- Bandwidth is FWHM of the power spectrum
- Equivalently, if $\mathrm{F}(\omega)$ represents **amplitude spectrum**, use -3dB

$$10\log_{10}\frac{P_2}{P_1} = 20\log_{10}\frac{A_2}{A_1}$$

$$10\log_{10}\frac{1}{2} = 20\log_{10}\frac{1}{\sqrt{2}} = -3dB$$

# Time-Bandwidth Relation

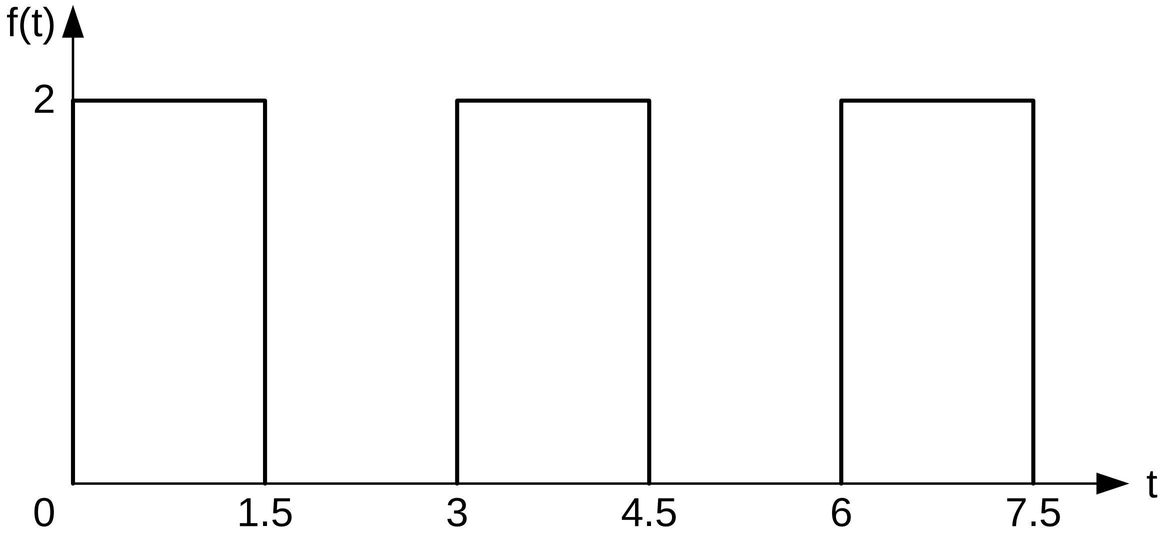
- Consequently, for any pulse shape, there is a fixed relationship between time-duration and bandwidth

- For a Gaussian the time-bandwidth product

$$\tau\Delta\omega = 0.44$$

- To preserve a pulse shape as it passes through an instrument, we must preserve the frequency content
- Applies to **any arbitrary input signal**

# Discussion question:



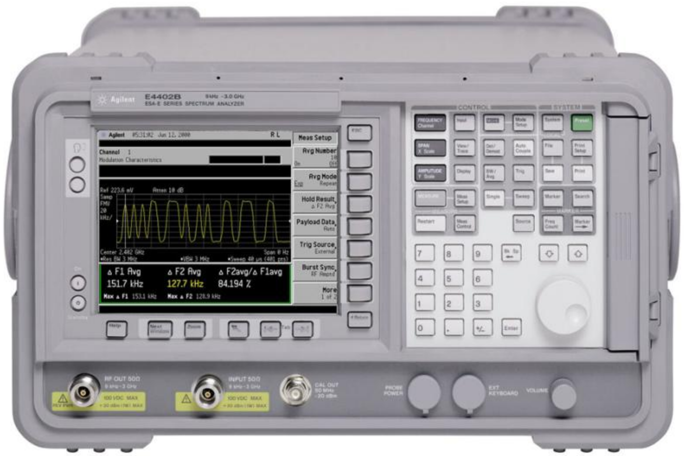

- How much does it cost to build an instrument to generate this waveform?

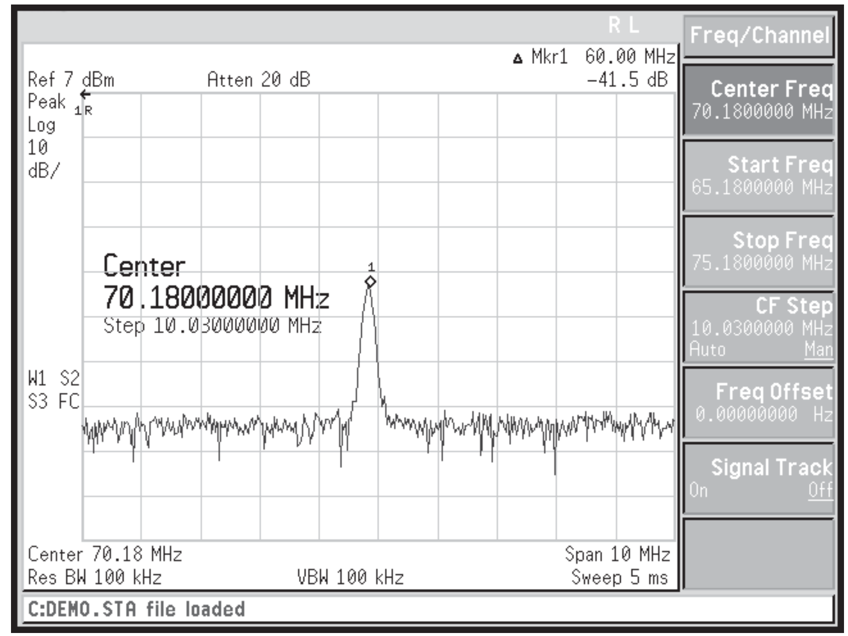
# Answer

- A mathematically perfect voltage output is not possible
- Bandwidth roughly ∝ cost
- We will always lose some frequencies and corrupt the signal

- Engineering: “The Art of Compromise”
  - Fidelity ∝ bandwidth but
  - Noise ∝ bandwidth and
  - Bandwidth costs money
  - Etc…
- We must analyse all the trade-offs when designing the instrument

# Example Instrument: Spectrum Analyser






- Calibrated measurement of signal power as a function of frequency
- Selectable bandwidth
  - equivalent to frequency-resolution, here 100 kHz

# Key-Points

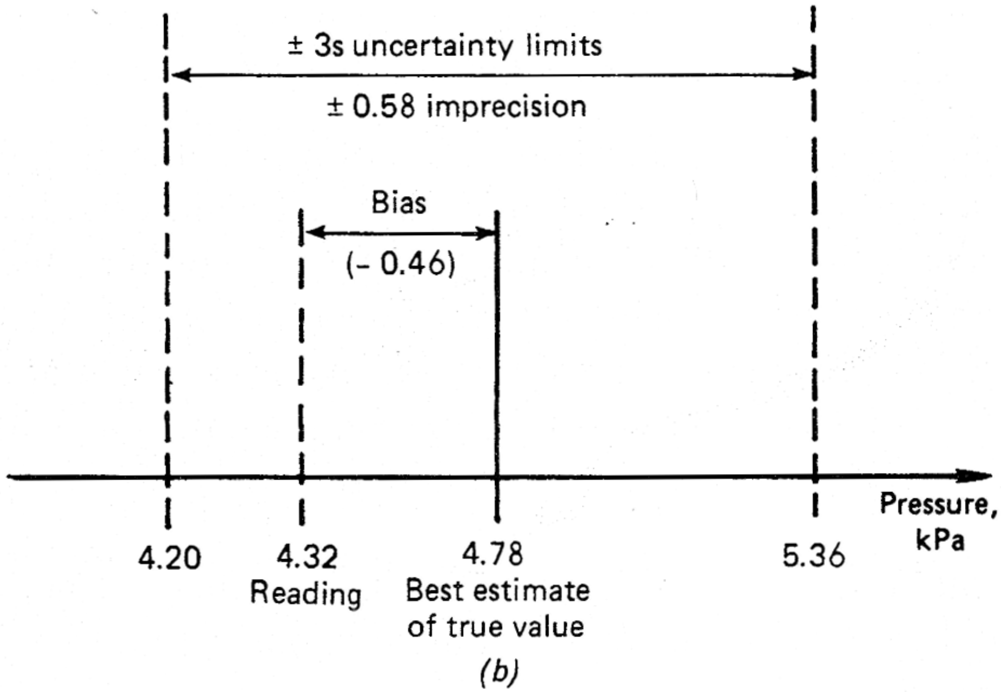
- **Mathematical** representations of signals tend to have infinite bandwidths ($\Delta\omega \to \infty$)
- **Real** signals tend to have very high $\Delta\omega$
- **Instruments** tend to have rather limited $\Delta\omega$
  - Either inherent or
  - Deliberate for
    - Noise reduction or
    - Stability

- Our signal is a physical measureable
- It is important to understand how finite bandwidth modifies the signal

# Part 2 of 4
# Instrument Characteristics and Calibration

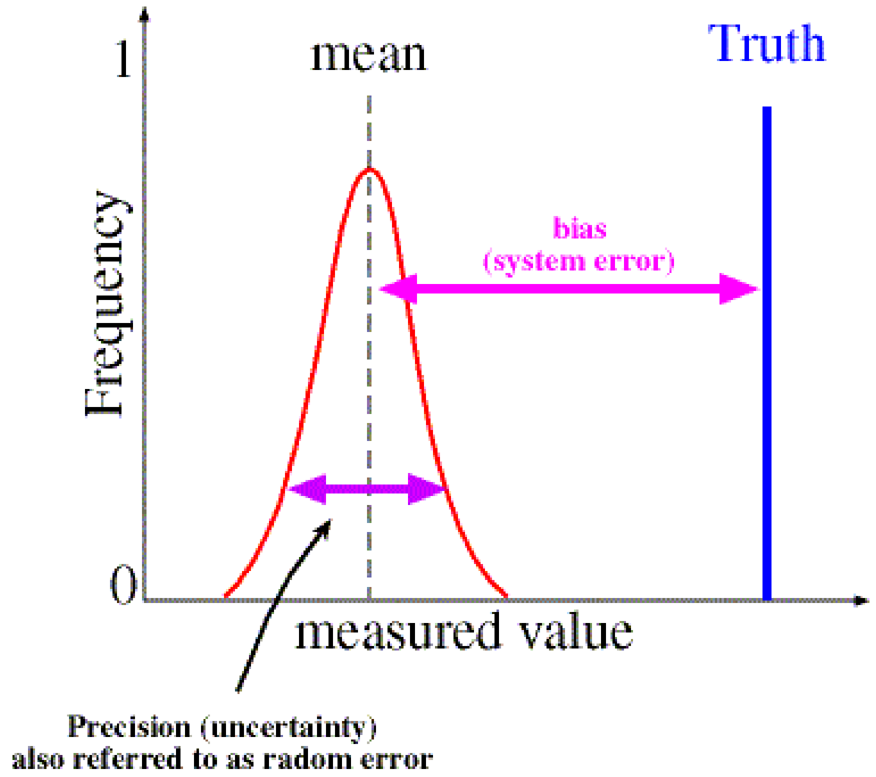
- We wish to make a measurement $f(t)$
- True value is **unknowable**
- Aim is to minimise our systematic and random errors
- Even the best instrument is only as good as the calibration standard or **reference measurement**

# Random Errors



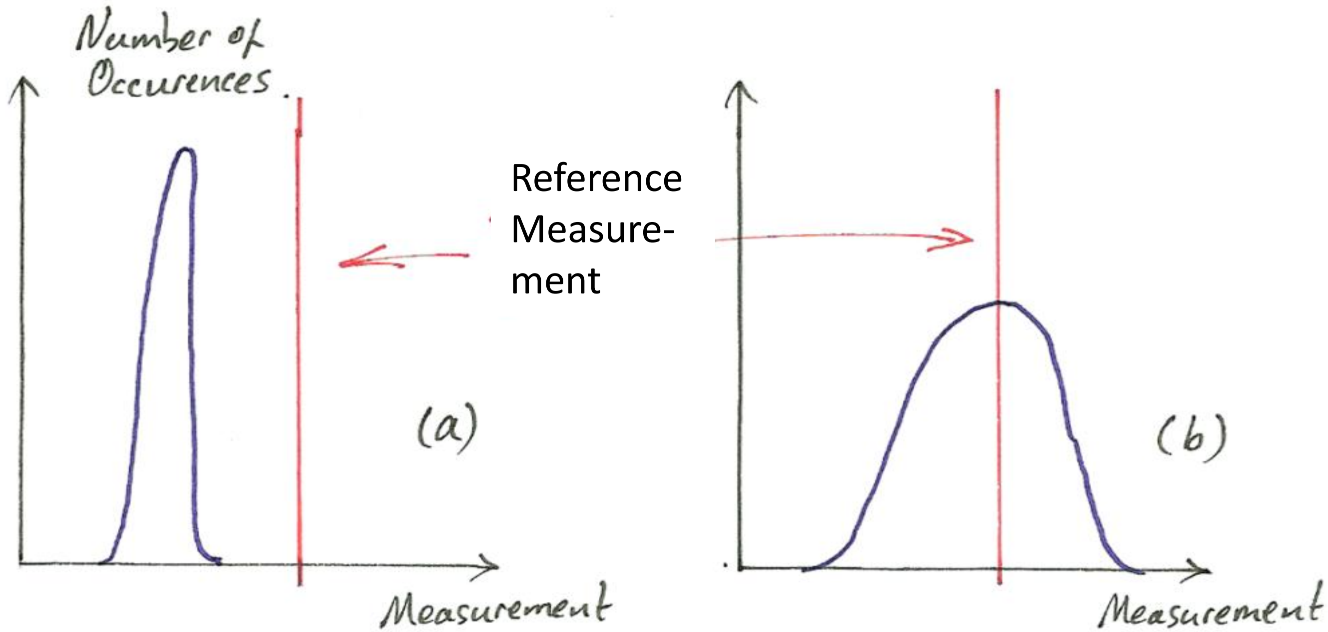

- Mean is best estimate of True Value
- Control random errors by
  - Design
  - Averaging to reduce measurement noise

# Systematic Error




- “Truth” ≡ Reference Measurement
- “Bias” ≡ Systematic Error
- “Precision” ≡ Std Deviation of distribution

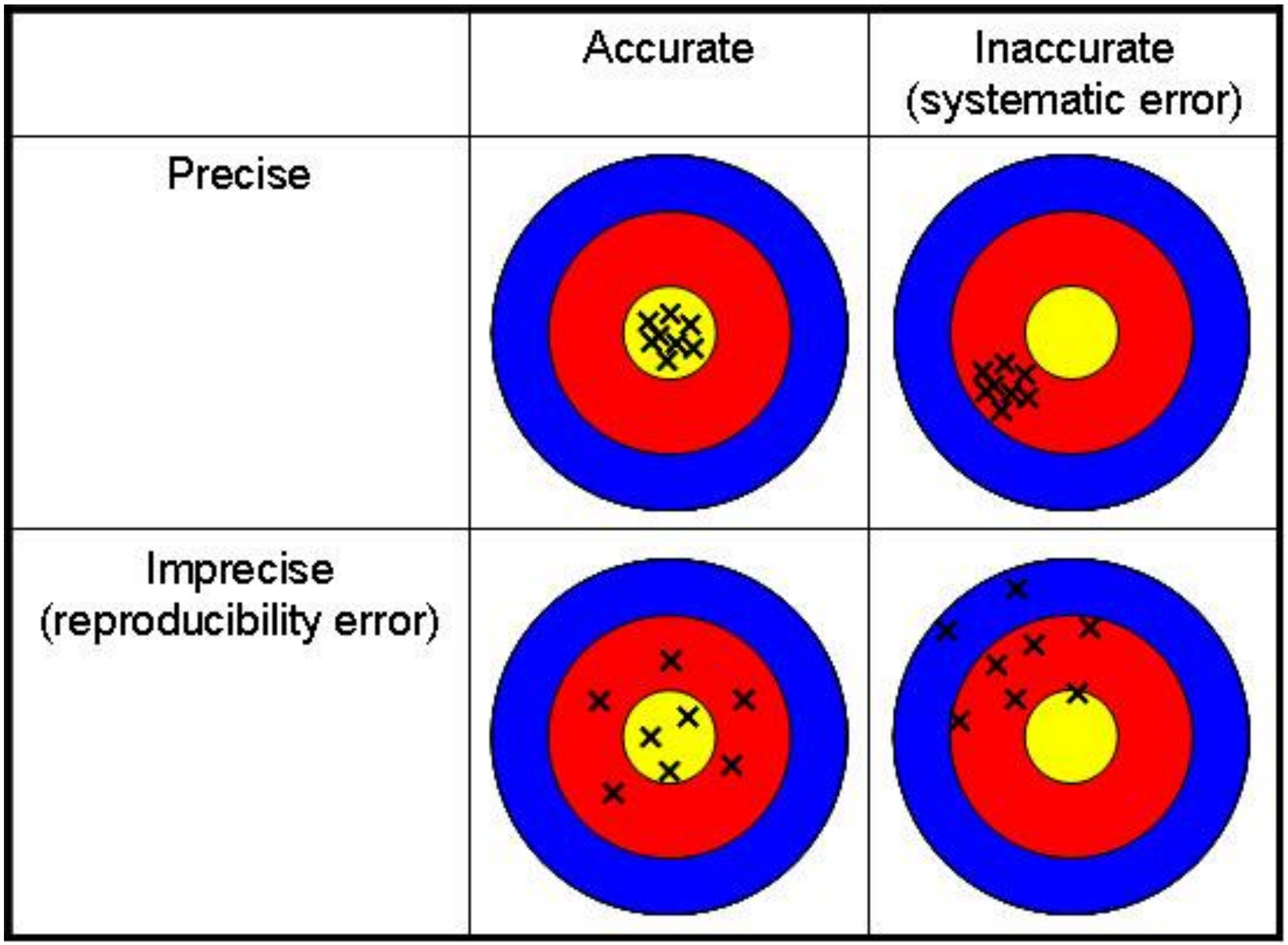
# Precision is not Accuracy




High Precision
Low Accuracy
(Poor calibration)

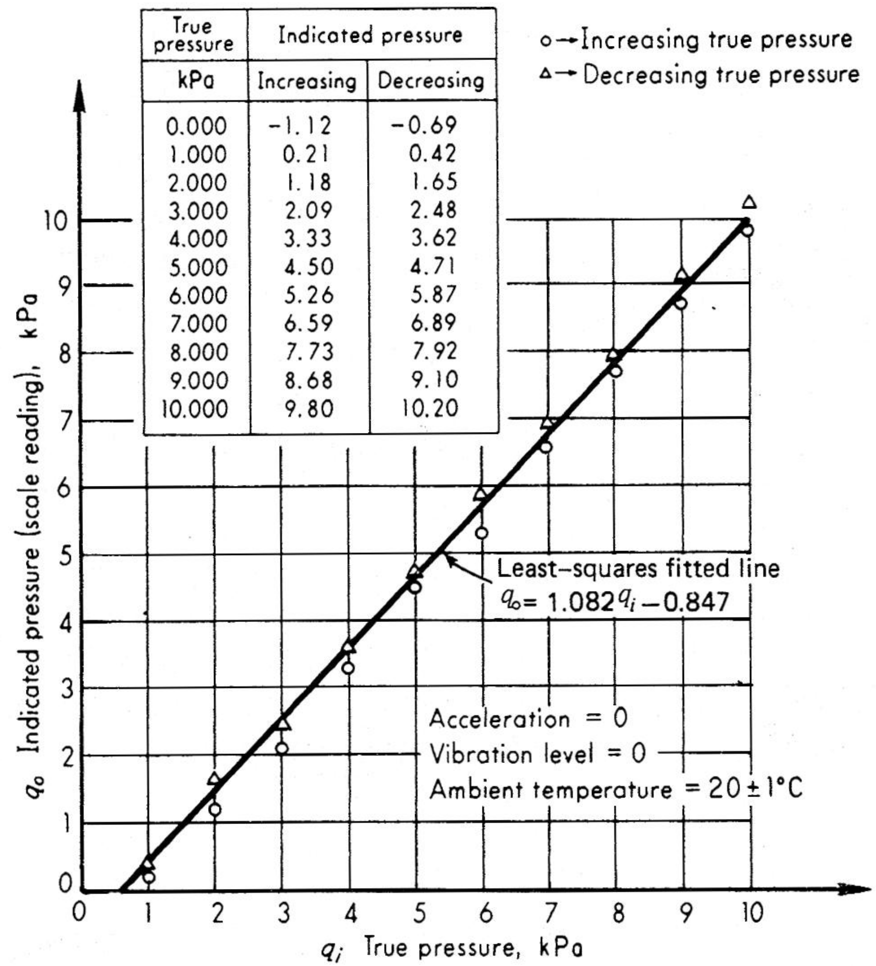
Low Precision
High Accuracy
(Bias well controlled)

# Cartoon Version



# Instrumental Effects

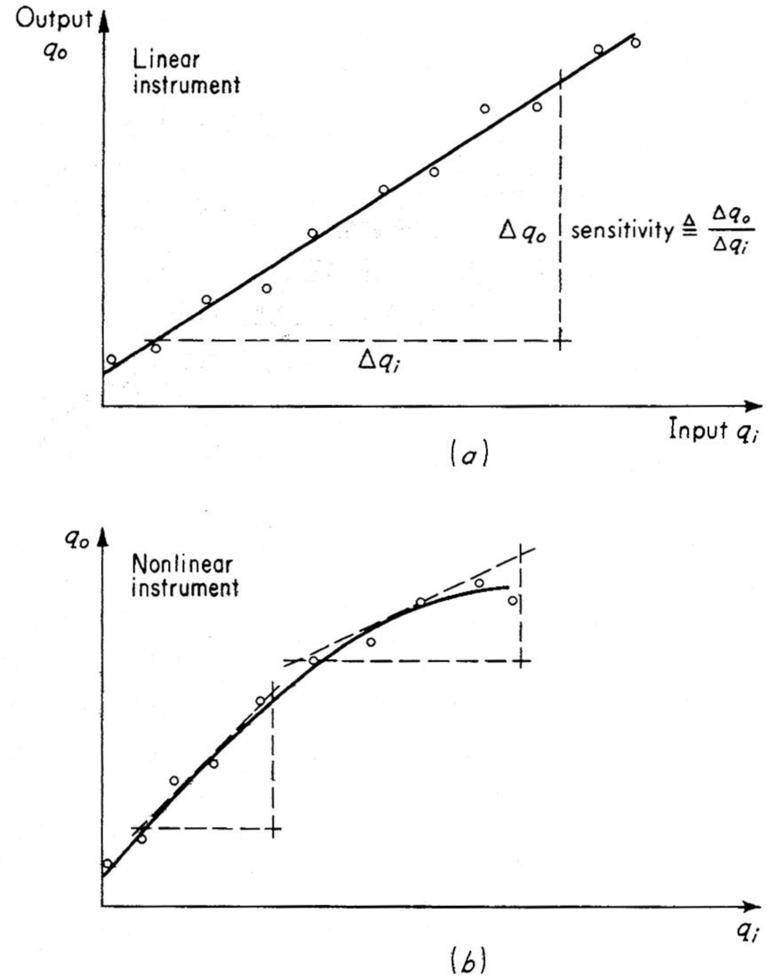
- Calibration: Comparison with reference measurement
- Quantify
  - Linearity
  - Dead-band
  - Hysteresis
  - Zero offset



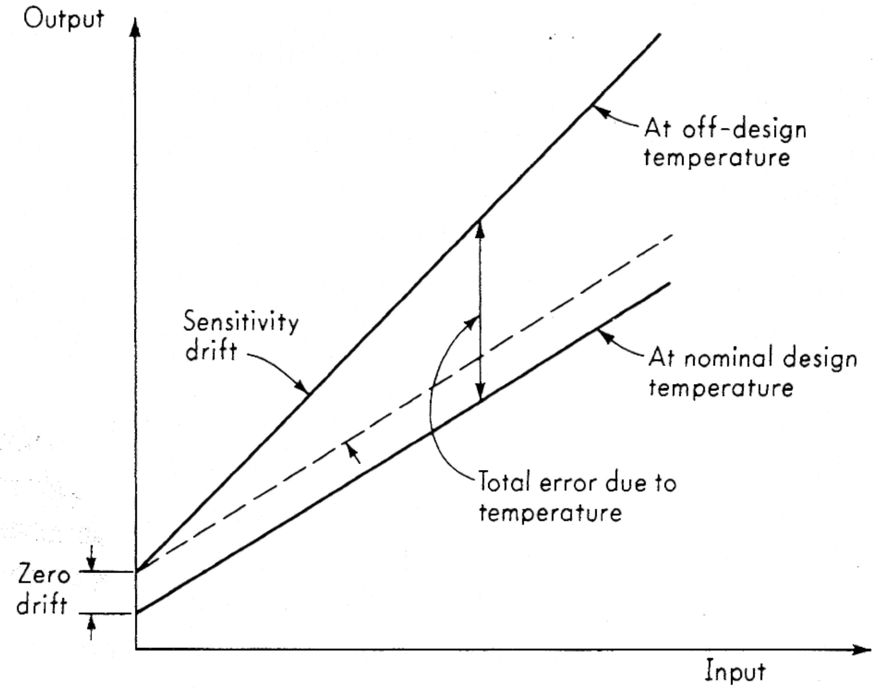
| True pressure kPa | Indicated pressure Increasing | Indicated pressure Decreasing |
|---|---|---|
| 0.000 | −1.12 | −0.69 |
| 1.000 | 0.21 | 0.42 |
| 2.000 | 1.18 | 1.65 |
| 3.000 | 2.09 | 2.48 |
| 4.000 | 3.33 | 3.62 |
| 5.000 | 4.50 | 4.71 |
| 6.000 | 5.26 | 5.87 |
| 7.000 | 6.59 | 6.89 |
| 8.000 | 7.73 | 7.92 |
| 9.000 | 8.68 | 9.10 |
| 10.000 | 9.80 | 10.20 |

# Linearity & Zero Offset

- Offset can be removed
- Linearity more pernicious
- Causes harmonic distortion for AC measurements
  - Minimise as highest priority
  - Limit operating range to linear regime
  - Use feedback



# Uncontrolled External Input




- Temperature-dependent sensitivity and offset
- Other environmental considerations
  - Pressure, acceleration, vibration, illumination
  - Drift, ageing (electronic systems)
  - Wear (mechanical systems)

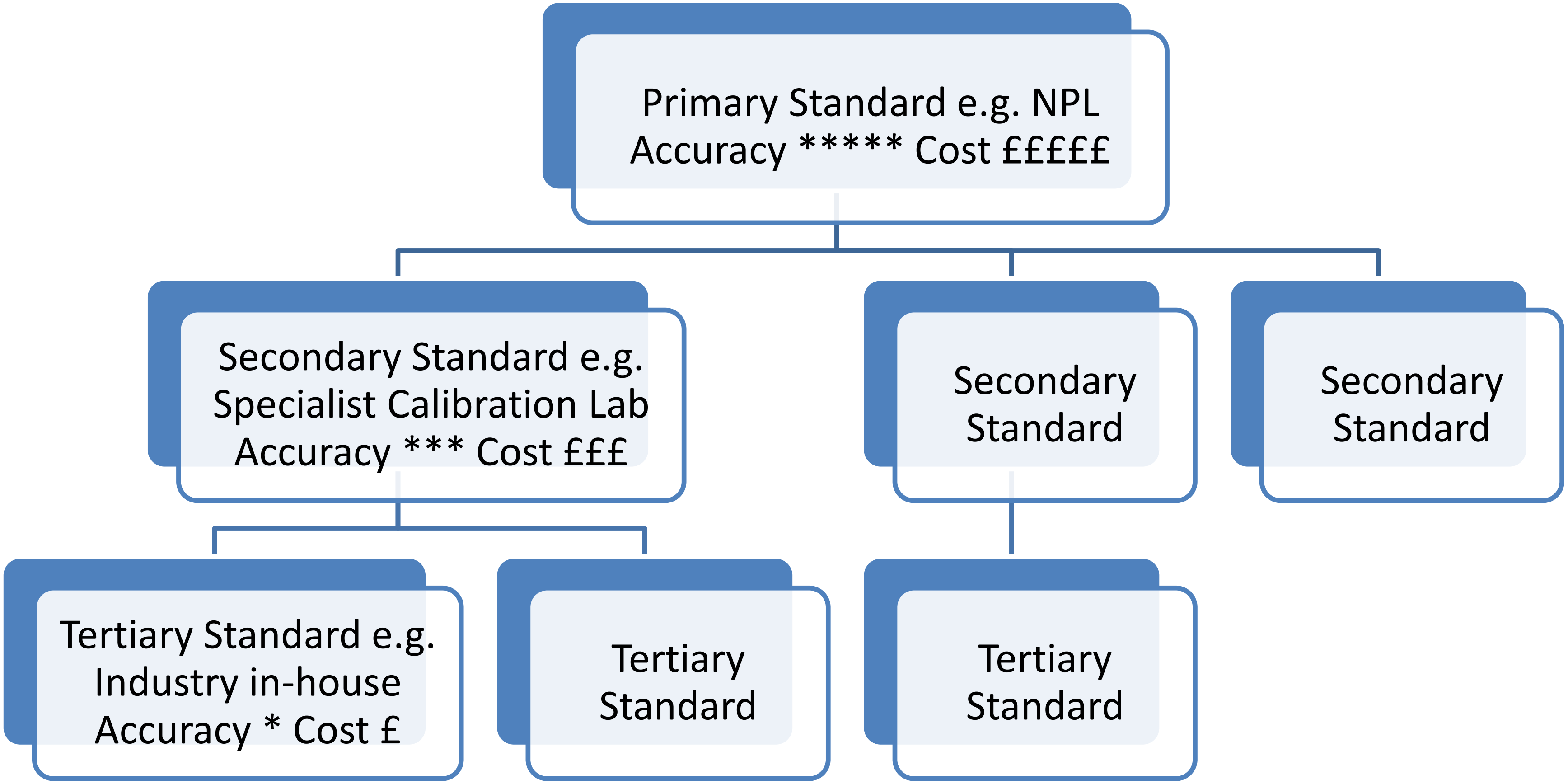
# Calibration Principle

- Compare against reference measurement with other input factors controlled / constant
  - Cover parameter space
  - Adjust external factors such as temperature
  - Multiple calibration curves

# Calibration Principle

- Helmholtz coils null Earth's field and apply test B
- Temperature-controlled Box houses Device Under Test
- Reference magnetometer mounted **outside** box

# Calibration Hierarchy

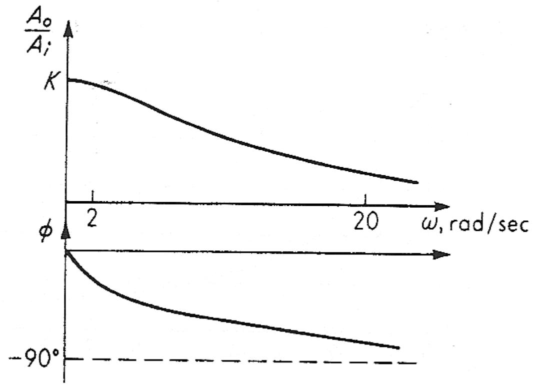



Our Reference Magnetometers are calibrated routinely by Ultra Electronics who are ISO 9000 accredited. Their equipment calibration is traceable back to national standards. Each comparison loses some accuracy but the error is bounded and known.

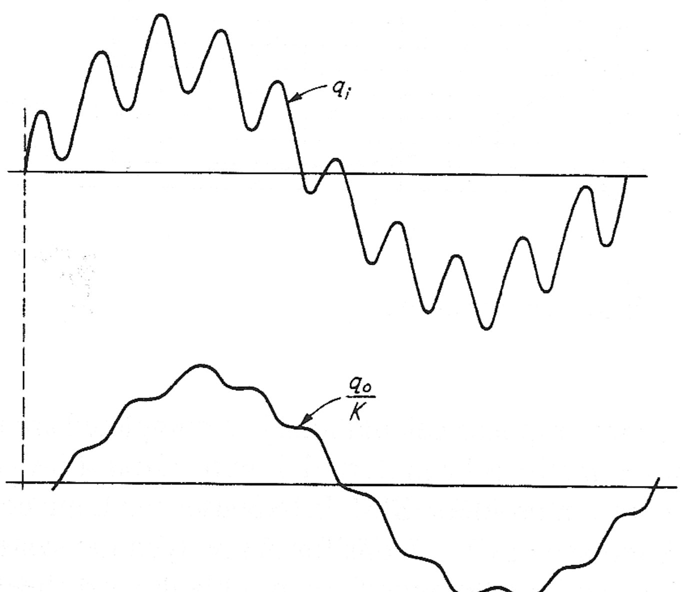
# Dynamic Calibration

- So far, we just considered **Static Calibration**
- Generally our measureable is a function of time
- For **Dynamic Calibration** measure
  - Frequency Response
    - Input is a sinusoid, swept across the bandwidth
  - Transient Response
    - Input is an impulse, step or ramp

# Frequency Response




(a)




(b)

- Bode Plot
  - Amplitude response
  - Phase response

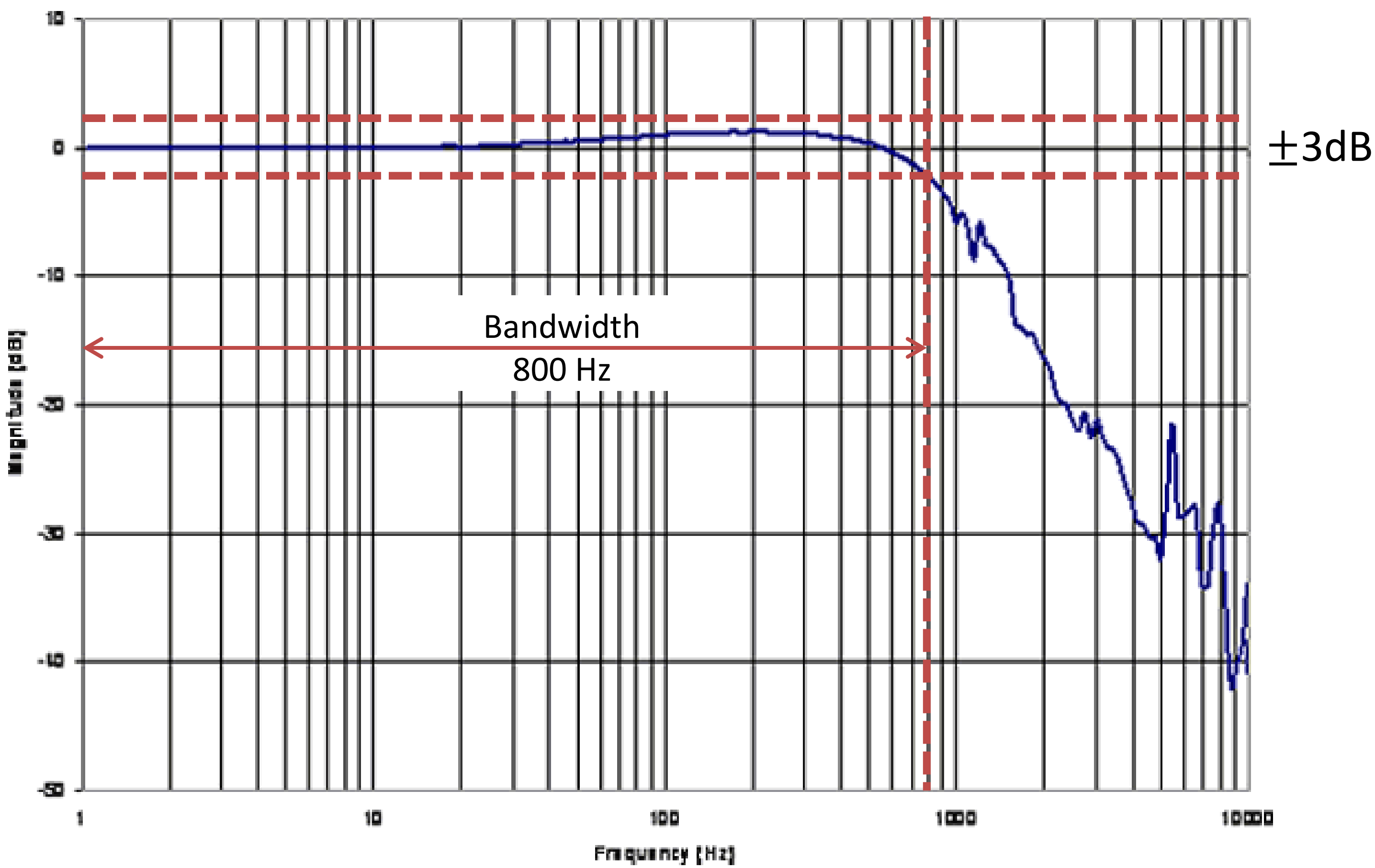
- The **transfer function** is

$$Gain(\omega) = \frac{Output(\omega)}{Input(\omega)}$$

- Gain is
  - Complex
  - Frequency dependent

# Practical Definition of "Instrument Bandwidth"

The range of frequencies for which output is within $\pm$3dB of the nominal Gain



# Transient Response

- Impulse $\delta(t)$ response
  - Stimulates all frequencies simultaneously
  - Can be used for direct experimental determination of the transfer function
  - Difficult (impossible!) to generate
- Step $u(t)$ response
  - More physically realisable

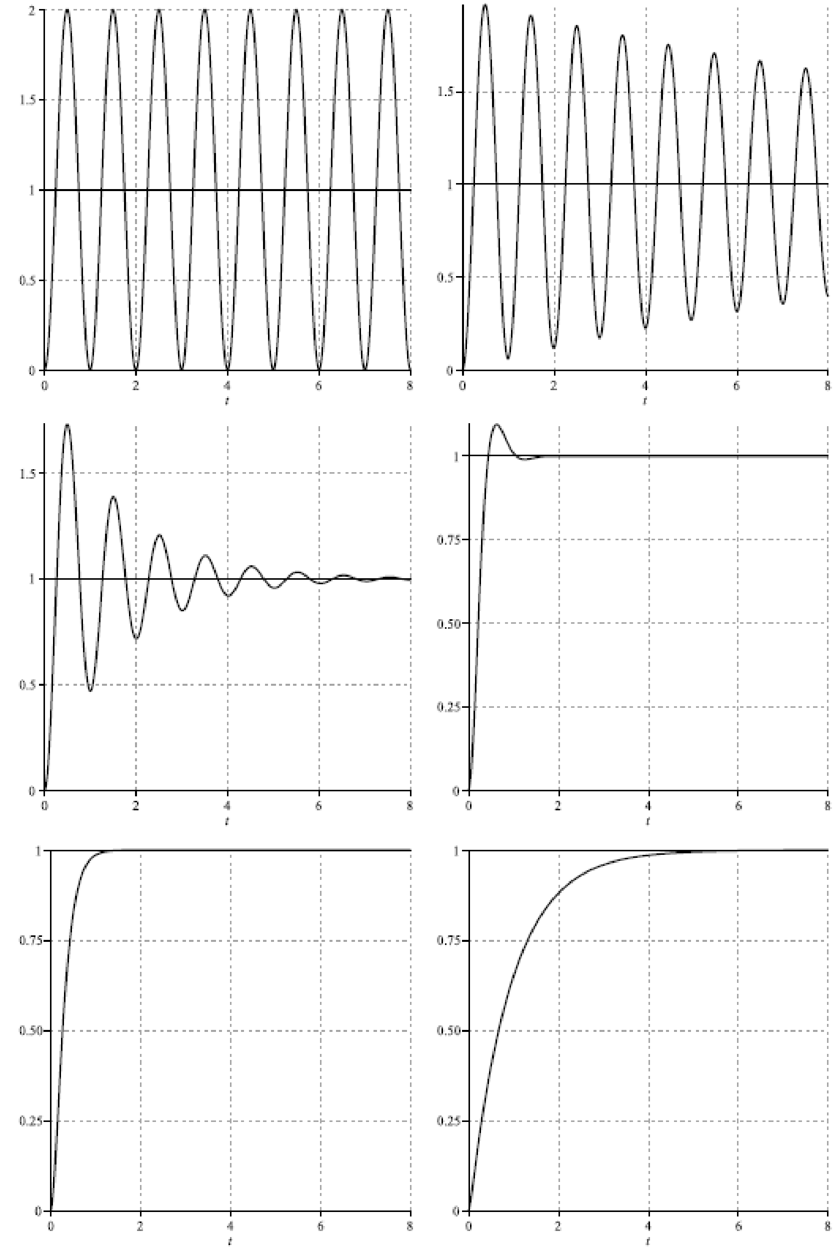
# Step Response for a 2nd order linear dynamic system

- Where the instrument response is modelled as a 2nd order differential equation

$$x(t) = a_0 y(t) + a_1 \frac{dy(t)}{dt} + a_2 \frac{d^2 y(t)}{dt^2}$$

Input Response

- Typical of **many** electrical, mechanical, thermal etc measurement systems
- Panels show instrument response as a function of time for cases no damping through to heavy damping
- Critical damped case (panel 4) gives optimum balance between delay and oscillation



# Consequences

- Instrument transient response can be characterised by
  - “Rise-time”
  - “Over-shoot”
  - “Settling-time”
- Too much damping kills-off frequency response
  - Reduces instrument bandwidth

# Key Points

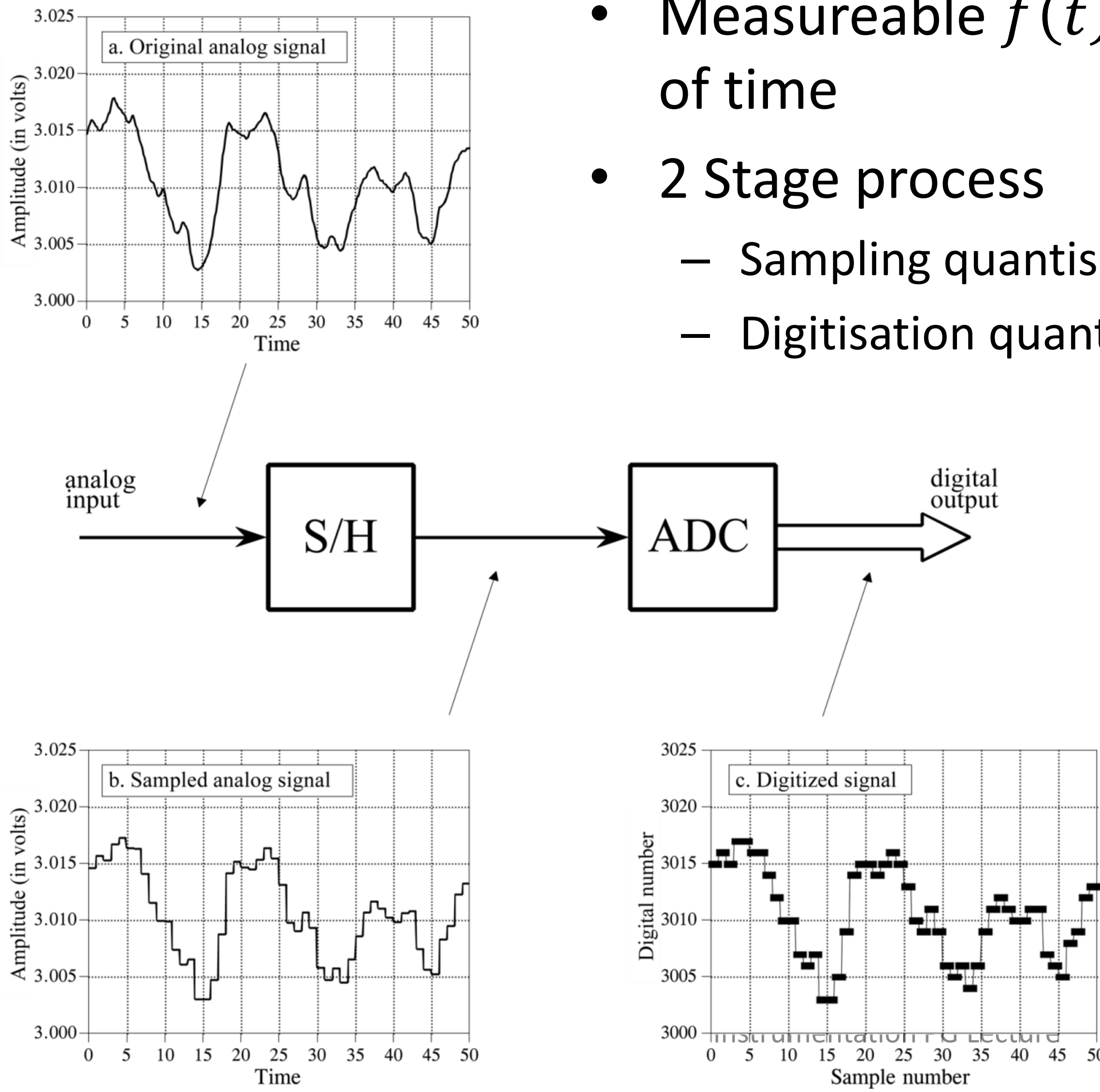
- The instrument is only as good as the reference measurement to which it is compared
- Linearity is a prime consideration in instrument design
- Systematic error is acceptable if quantifiable
- Responsiveness in the time-domain must be traded-off against bandwidth

# Part 3 of 4
# Digital Signals

- Our measureable is a continuous function of (usually) time
- Data is always sampled
- The samples are always digitised
- Each step we **lose some information**
  - we can model this as adding noise to the underlying signal

# Sampling and Digitisation

- Measureable $f(t)$ is a continuous function of time
- 2 Stage process
  - Sampling quantises $t$
  - Digitisation quantises $f(t)$



# Frequency Content of Sampled Signals

- Sampled signal (time domain)

$$f_s(t) = \sum_{n=-\infty}^{\infty} f(nT_s)\delta(t - nT_s)$$

$$T_s = \text{sampling interval}$$

- Sampled signal (frequency domain)

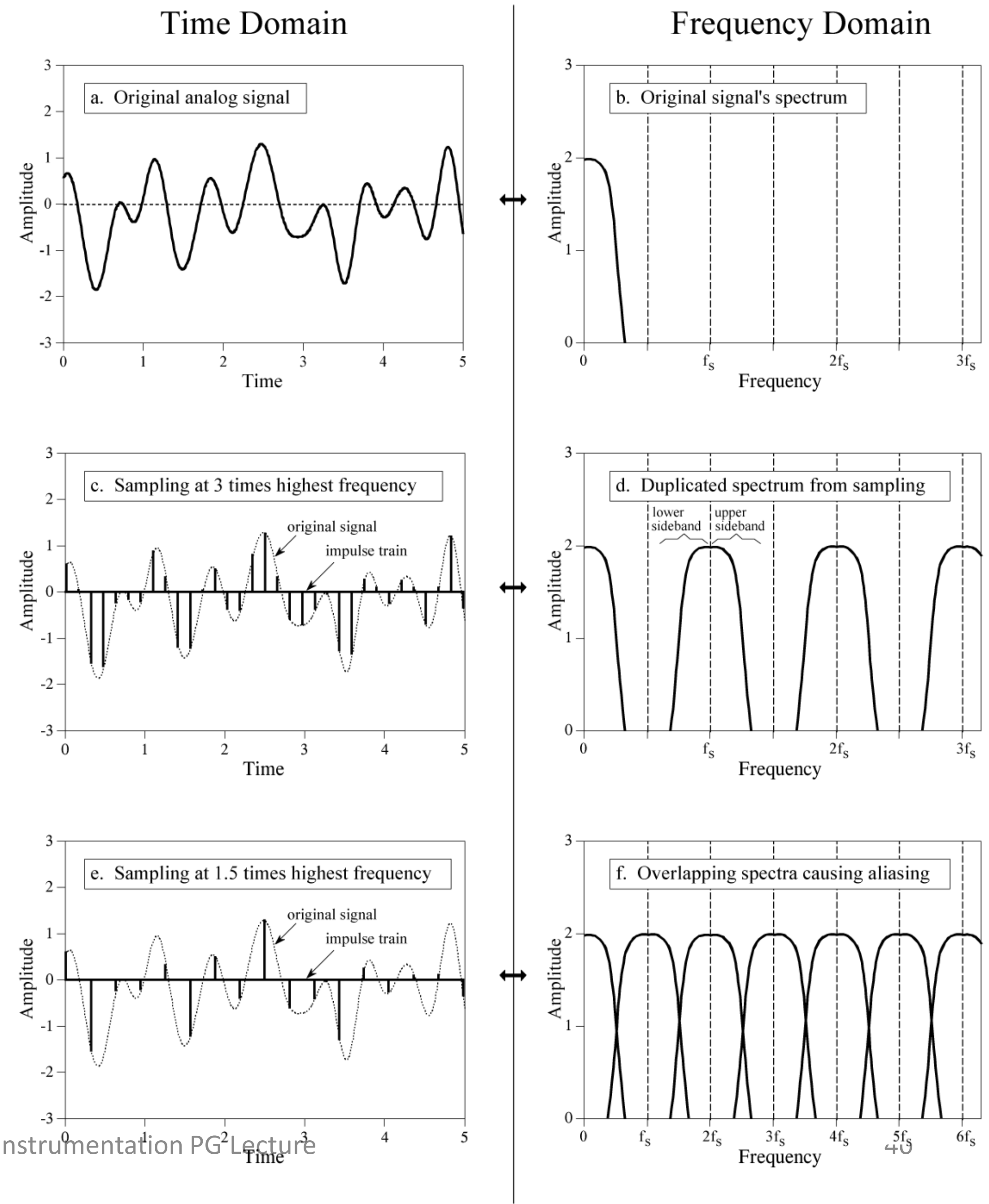
$$F_s(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} F(\omega + n\omega_s)$$

$$\omega_s = \frac{2\pi}{T_s}$$

- Consequences: Sampled signal repeats in the frequency domain

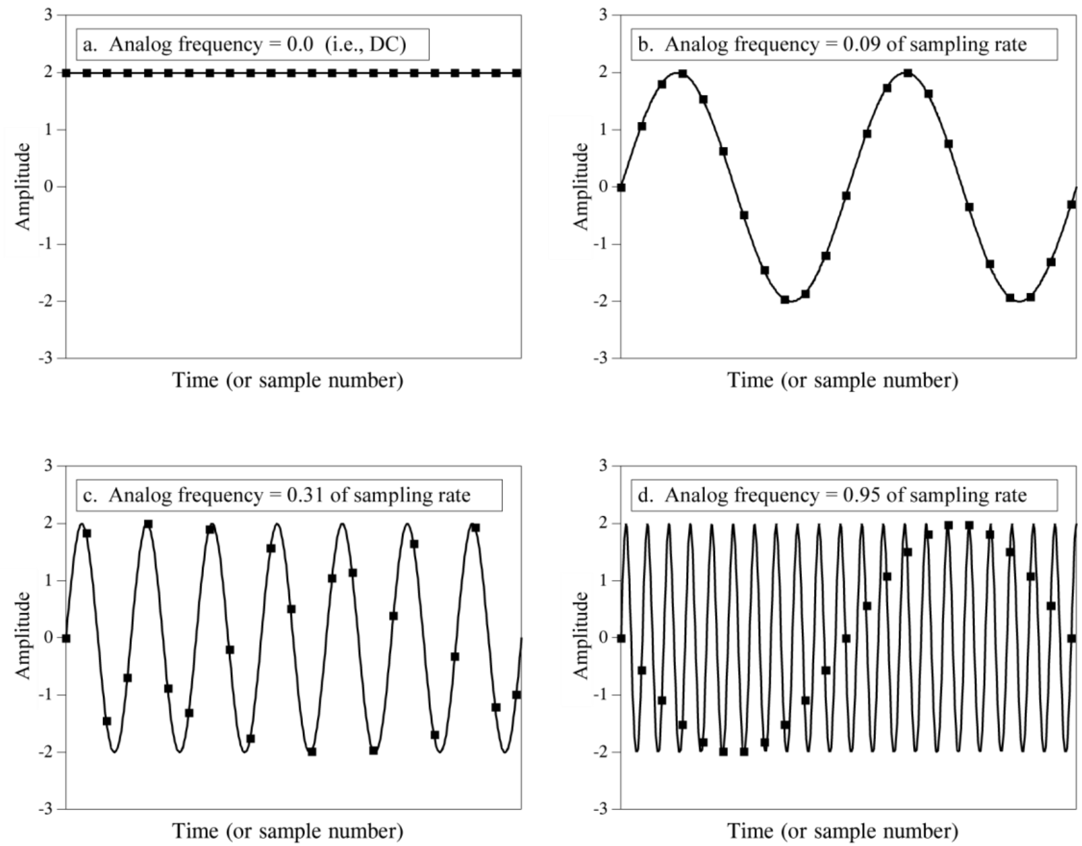
# Aliasing

- Overlapping spectra
- Higher frequency components of the signal are incorrectly represented as lower frequencies
- **Nyquist criterion (to avoid aliasing): Sampling frequency $> 2 \times$ highest frequency component in $f(t)$**

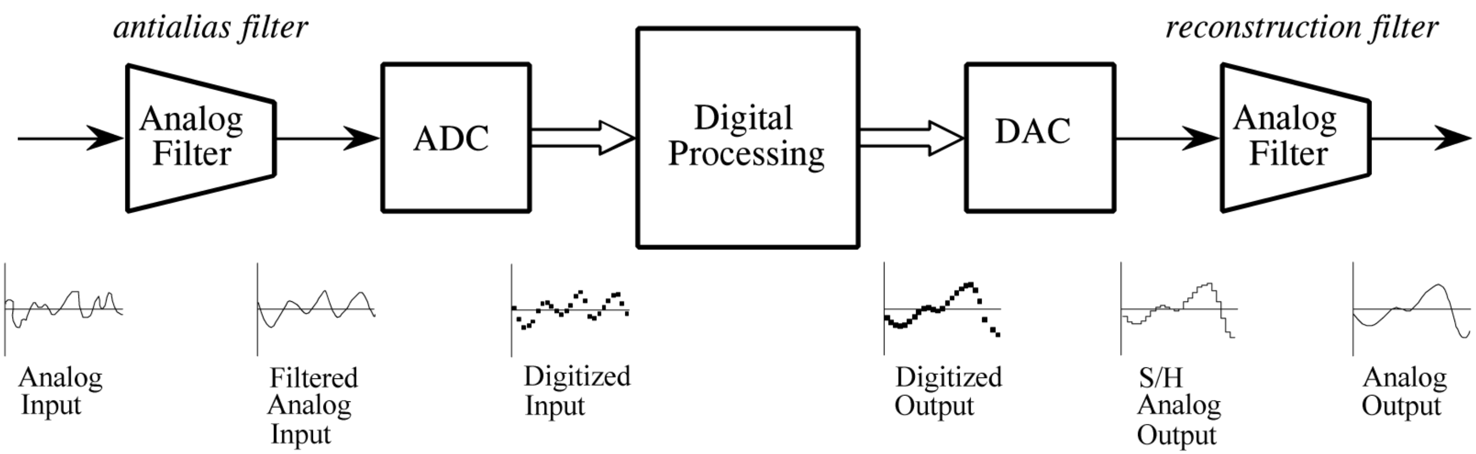


# Graphical Illustration of Aliasing

- Solid line: $f(t)$
- Dots: samples
- FFT will find lowest frequency sinusoidal fit to the dots



# Avoid Aliasing!
# High priority in instrument design



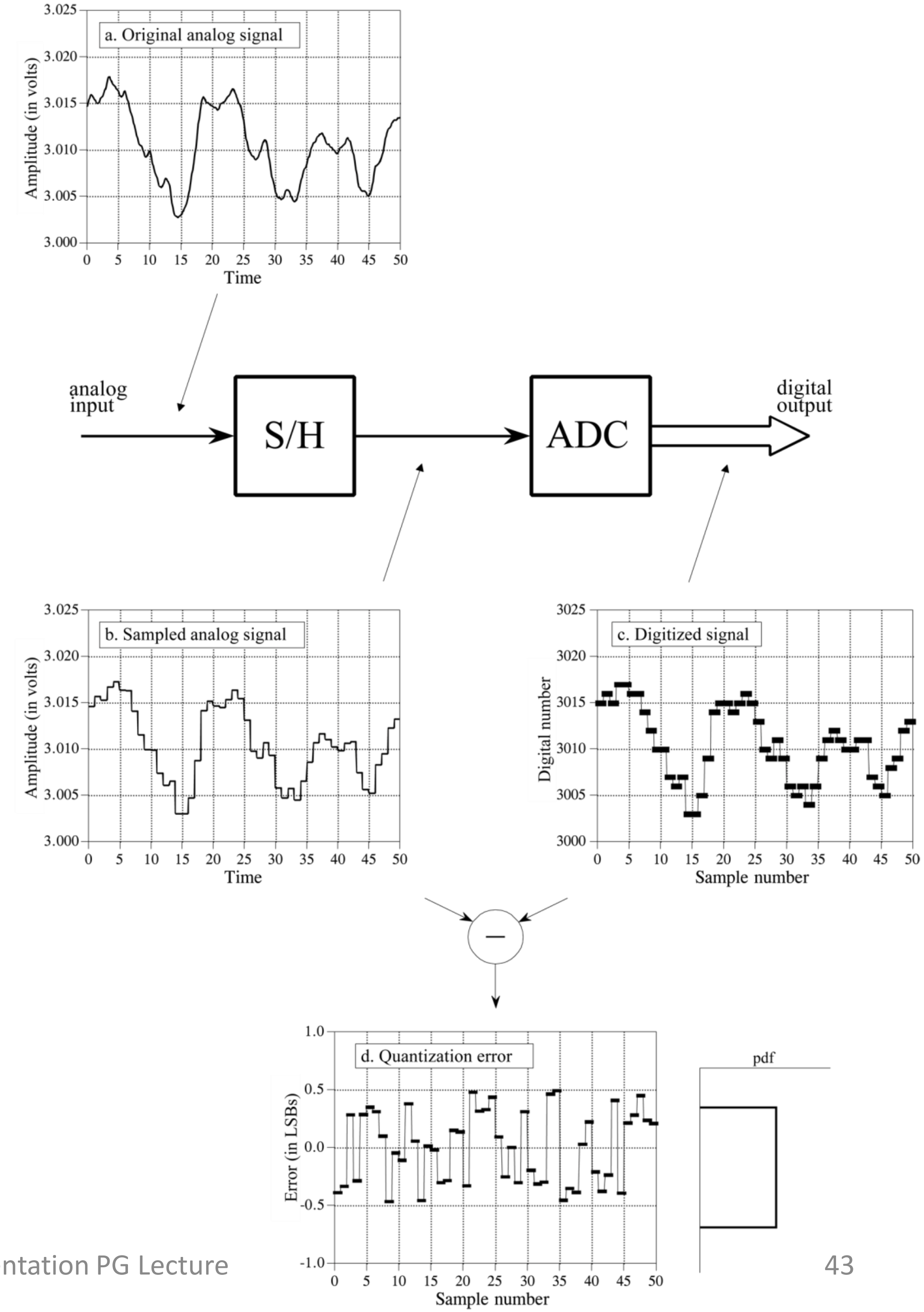

– Anti-alias filter
  - Filters the **analogue** signal
  - **Removes** frequencies higher than the Nyquist limit

# Digitised Signals

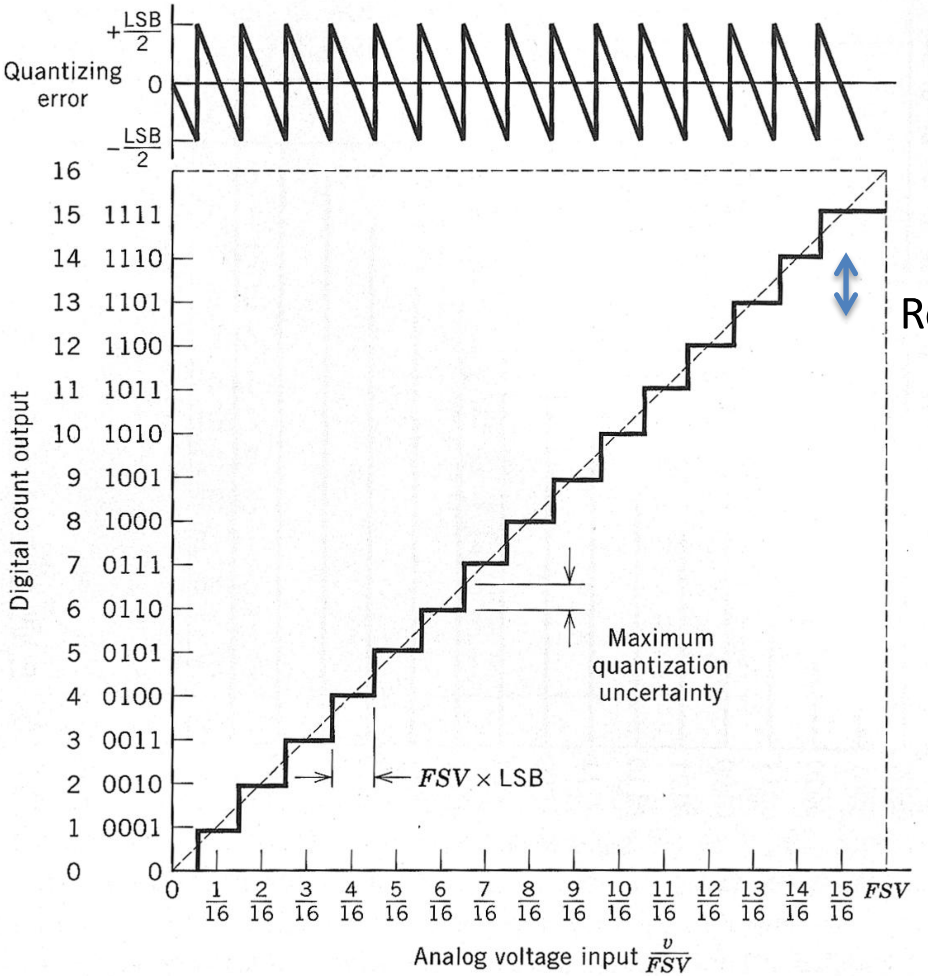
- Digitisation quantises $f(t)$
- Results in quantisation error

$$\sigma \sim \frac{1}{\sqrt{12}} LSB$$

- LSB $\equiv$ digital resolution



# Range and Resolution




Digital Resolution

Input Range

Dynamic Range

$$DR_{dB} = 20\log_{10}\left(\frac{\text{Range}}{\text{Resolution}}\right)$$

An $n$-bit ADC allows $2^n$ digital values

$$DR_{dB} = 20\log_{10}(2^n)$$

# Key Points

- Digital signals lose information which can never be recovered
- Avoid aliasing at all costs: obey the Nyquist criterion
- Know the quantisation error
- Digital resolution should be consistent with other stochastic processes such as noise or random error
  - Contributions from these processes add in quadrature

# Part 4 of 4
# Noise

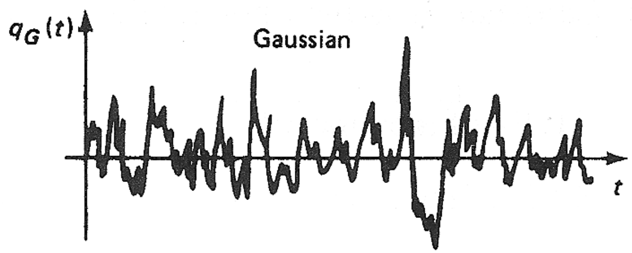
- Is usually the limiting factor in our measurement ability
- Comes from
  - The sensor (physics of the measurement)
  - The electronics
  - Digitisation
  - Interference

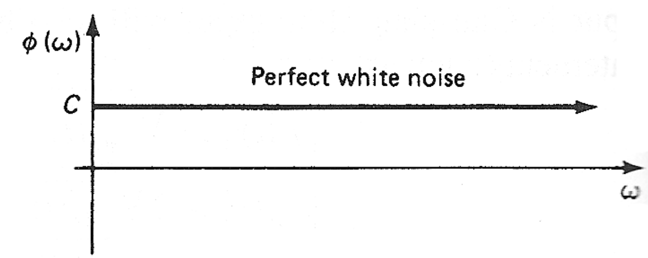
# Sources of noise in experimental data

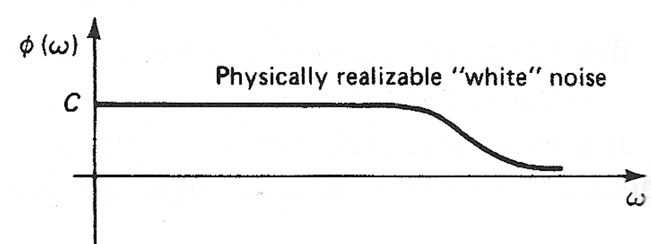
- **Total noise in measurement**
  - **Intrinsic Noise**
    - **Sensor physics**
      - E.g. Barkhausen noise from magnetic materials
    - **Sensor electronics**
      - Thermal noise
      - Shot noise
    - **Measurement noise**
      - Quantisation noise
      - Flicker or $\frac{1}{f}$ noise
  - **Extrinsic Noise**
    - **Sensor Pickup**
      - Environmental Interference e.g. magnetic sources
    - **Electronic Interference**
      - Conductive pickup through power/signal wires
      - Radiative pickup by Magnetic field (inductive) or Electric Field (capacitive)

# Noise comes from stochastic processes

- Can only be described statistically
- Amplitude probability function
  - Normal (Gaussian) for shot, thermal, flicker
  - Uniform (flat) for quantisation noise
- Power Spectrum
  - Flat (white) for thermal, shot
  - $\propto \frac{1}{f}$ (pink) for flicker



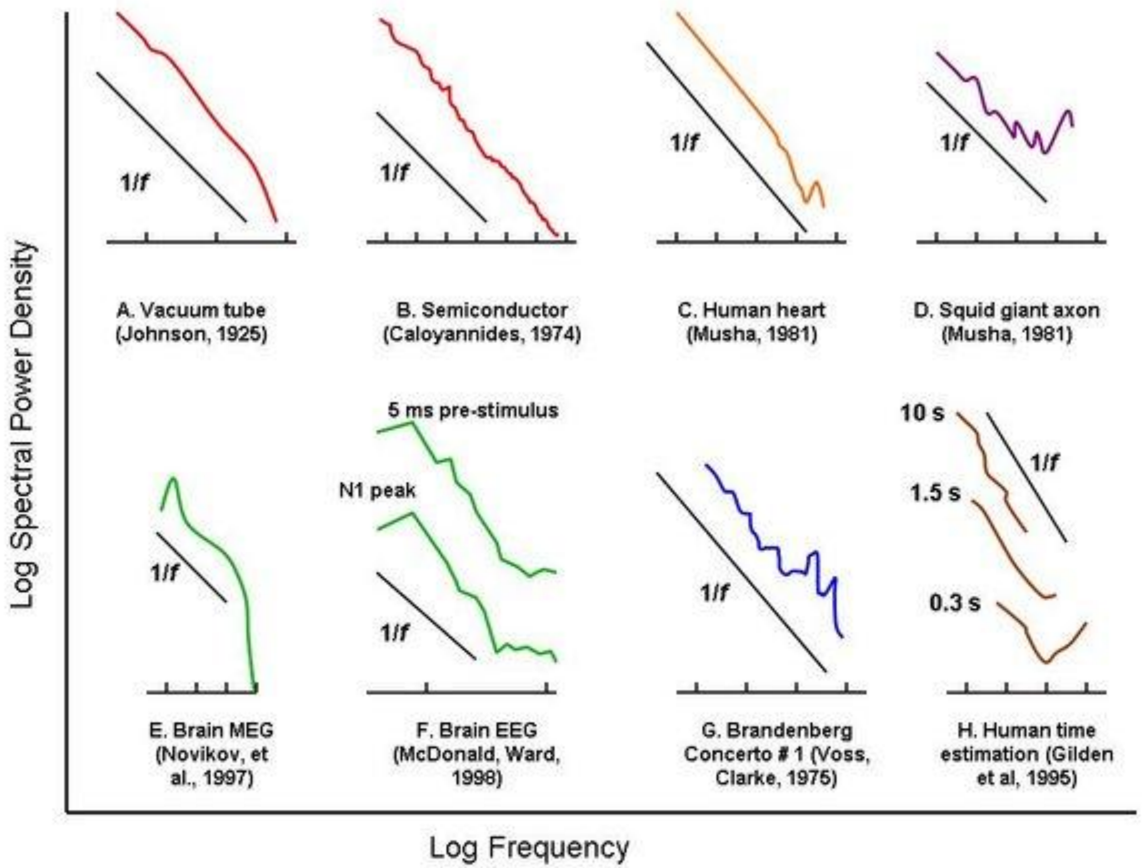
# Example: Thermal Noise in sensors and electronics

- Arises from the random thermal movement of conduction electrons
  - Function of temperature
- The RMS noise voltage measured with an instrument bandwidth B is

$$V_n = \sqrt{4k_B TRB}$$

- $T/°K$
- $R$ Resistance
- $B$ Bandwidth

# Flicker or $\frac{1}{f}$ noise



# Composite Noise Power Spectrum

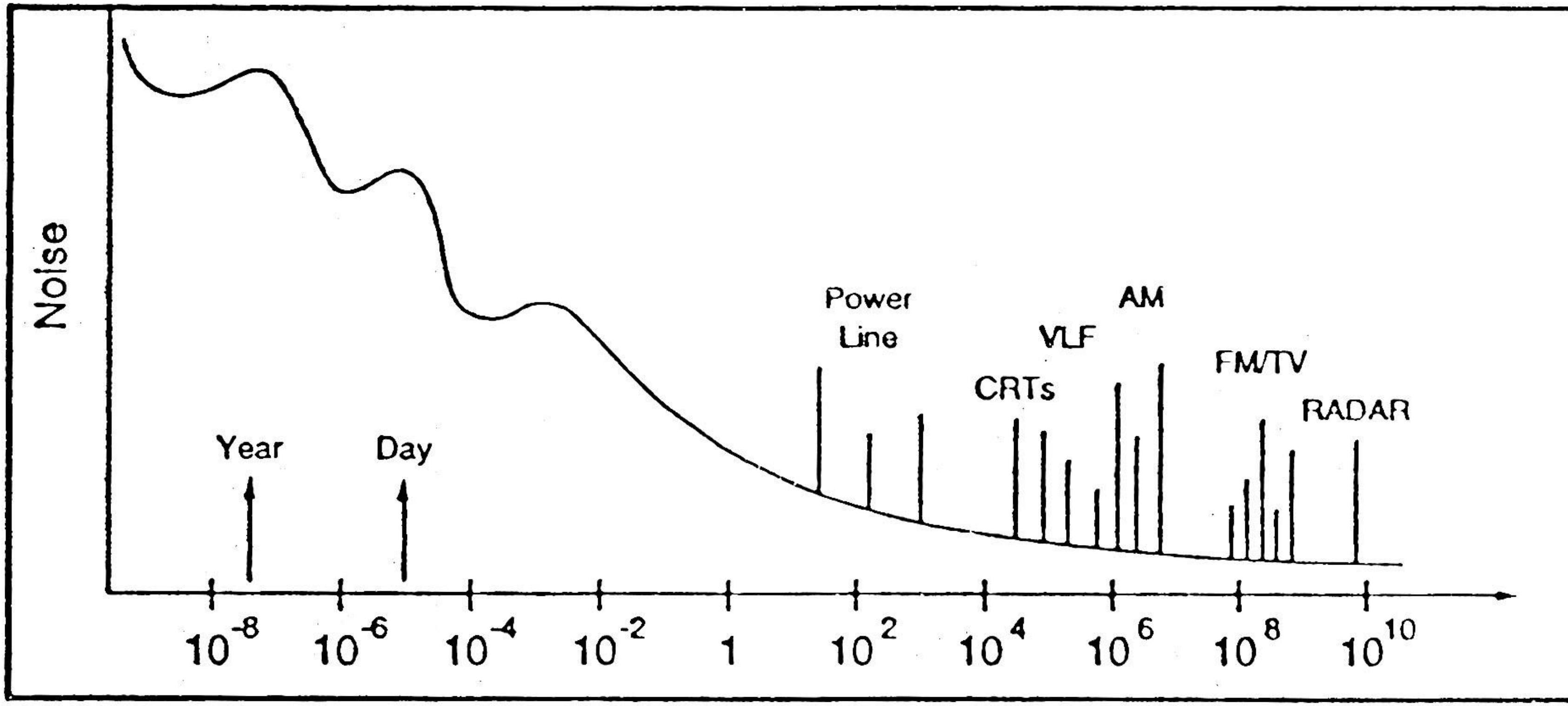
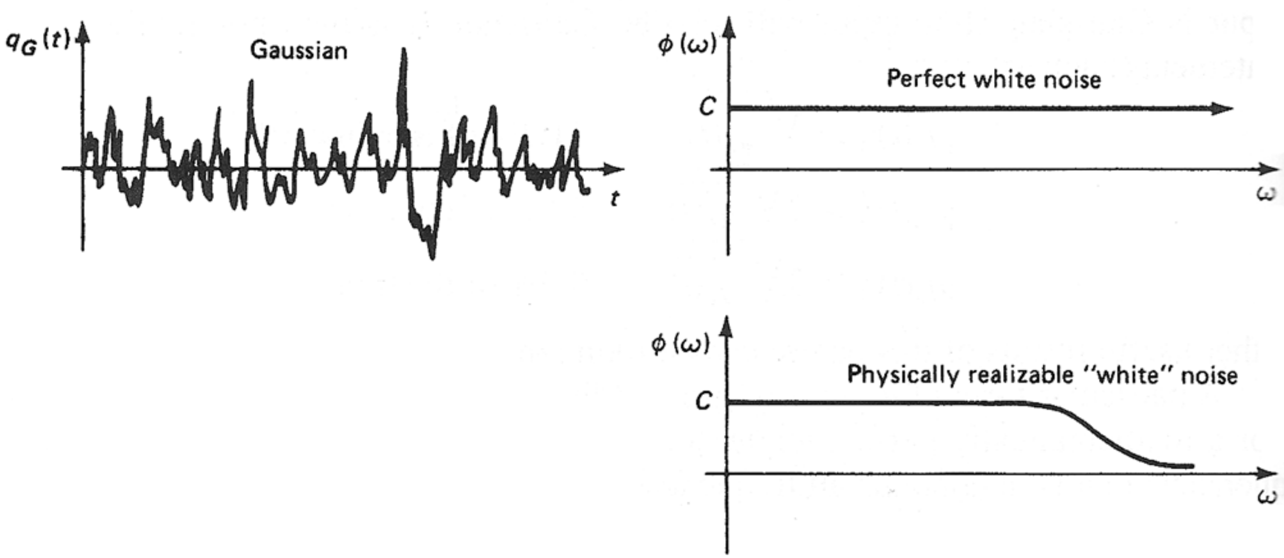



Figure 3: Simplified Noise Spectrum

# Consequences and Mitigation

- Noise is a function of physical parameters such as temperature, resistance, current but **always** bandwidth
- Reducing bandwidth reduces total noise measured
- Filter signal to remove unwanted frequency components



# Key Points

- Noise is inherent in ally physical processes and is a function of bandwidth
- Most noise reduction techniques work by limiting the bandwidth of the measurement
- We must balance this against the impact on signal fidelity

# References, sources and further reading


- Doebelin, E.O., Measurement Systems Application and Design (McGraw Hill, 2004)
- Poularikas & Seeley, Elements of Signals and Systems (PWS Kent, 1988)
- Smith, S. W., Scientist and Engineer’s Guide to Digital Signal Processing (Newnes, 2003) (also online at www.dspguide.com)