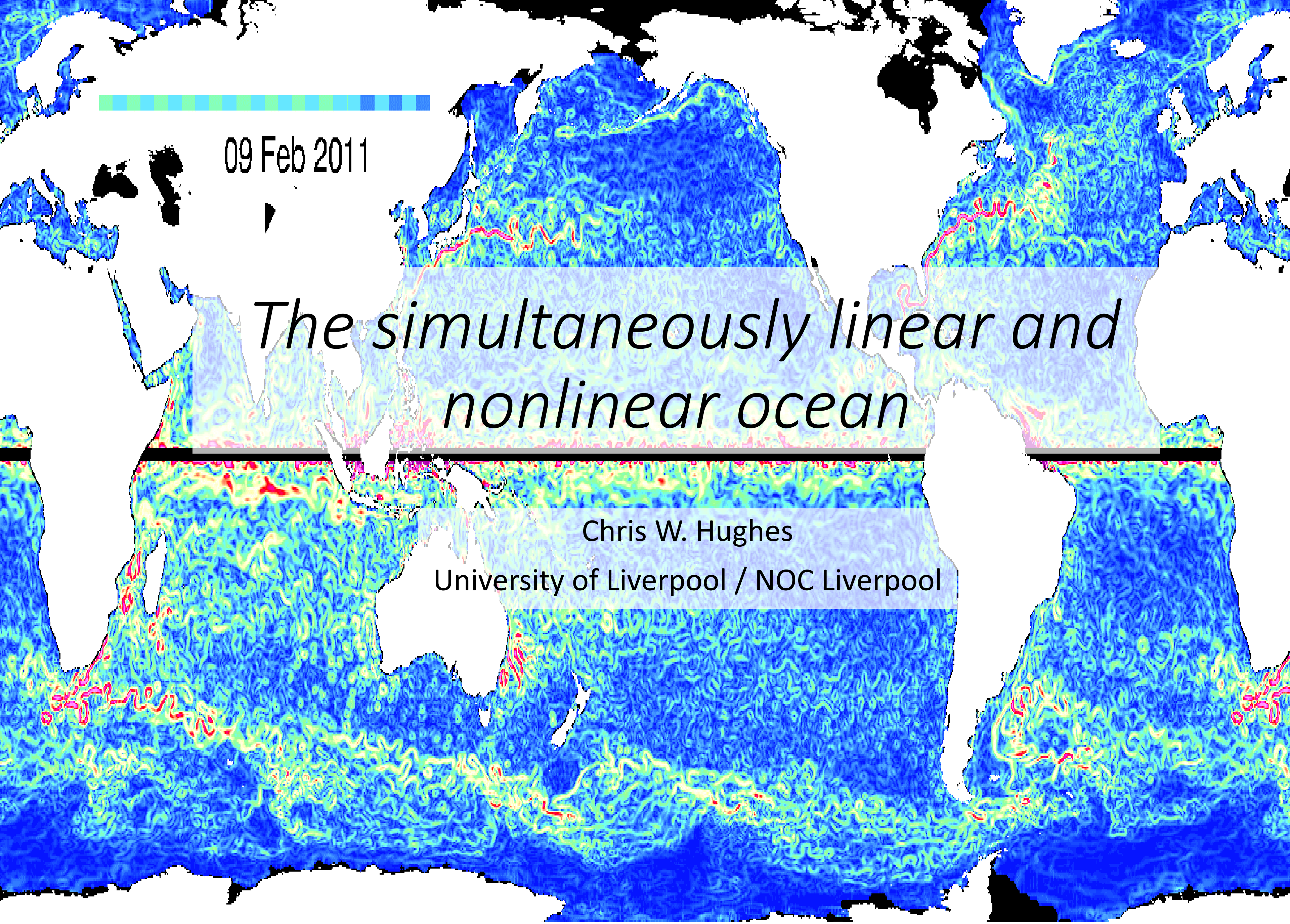
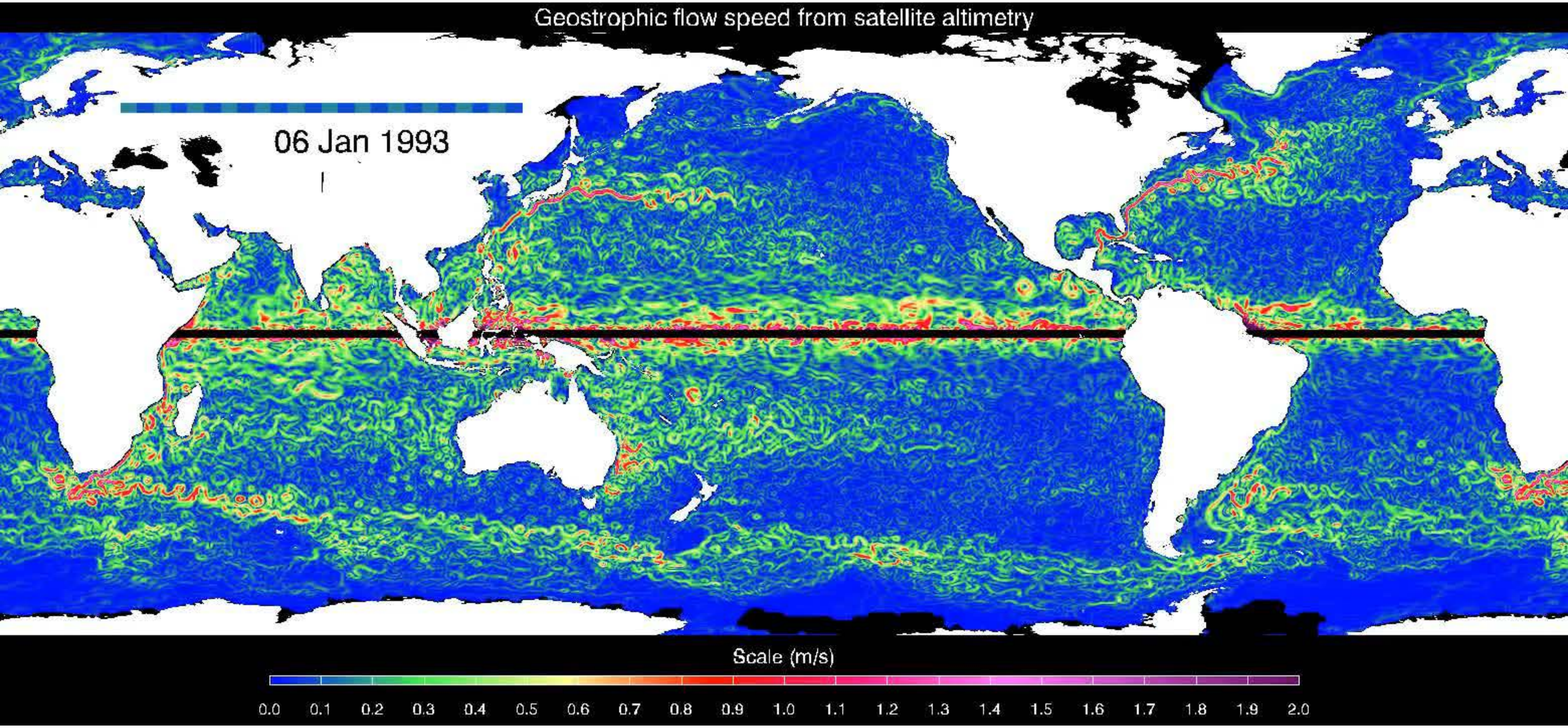


The simultaneously linear and nonlinear ocean

Chris W. Hughes
University of Liverpool / NOC Liverpool

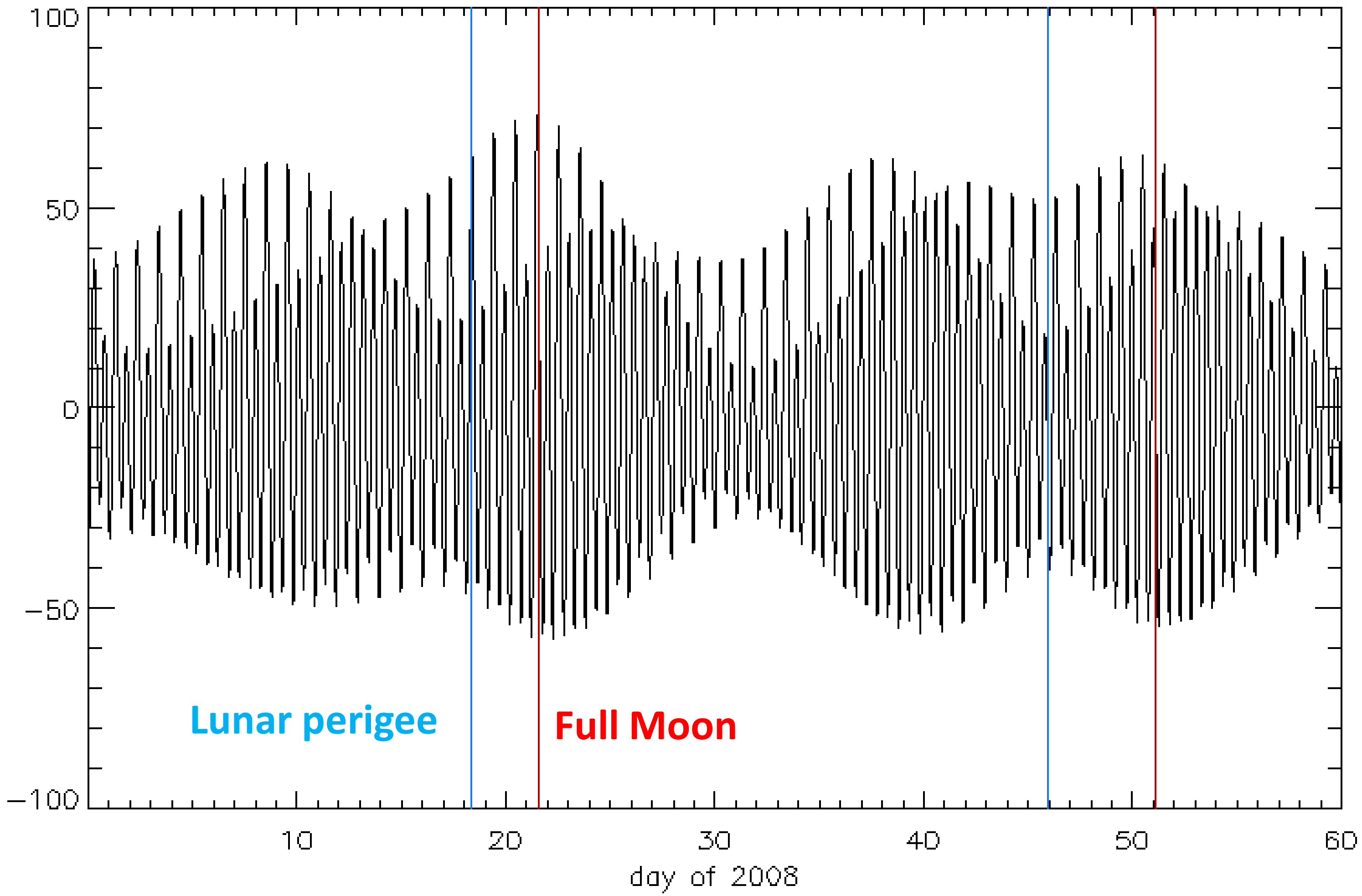
Geophysical fluid dynamics with a twist: in honour of Raymond Hide 30th April 2018

Ocean currents derived from satellite altimetry data (sea level measurements)



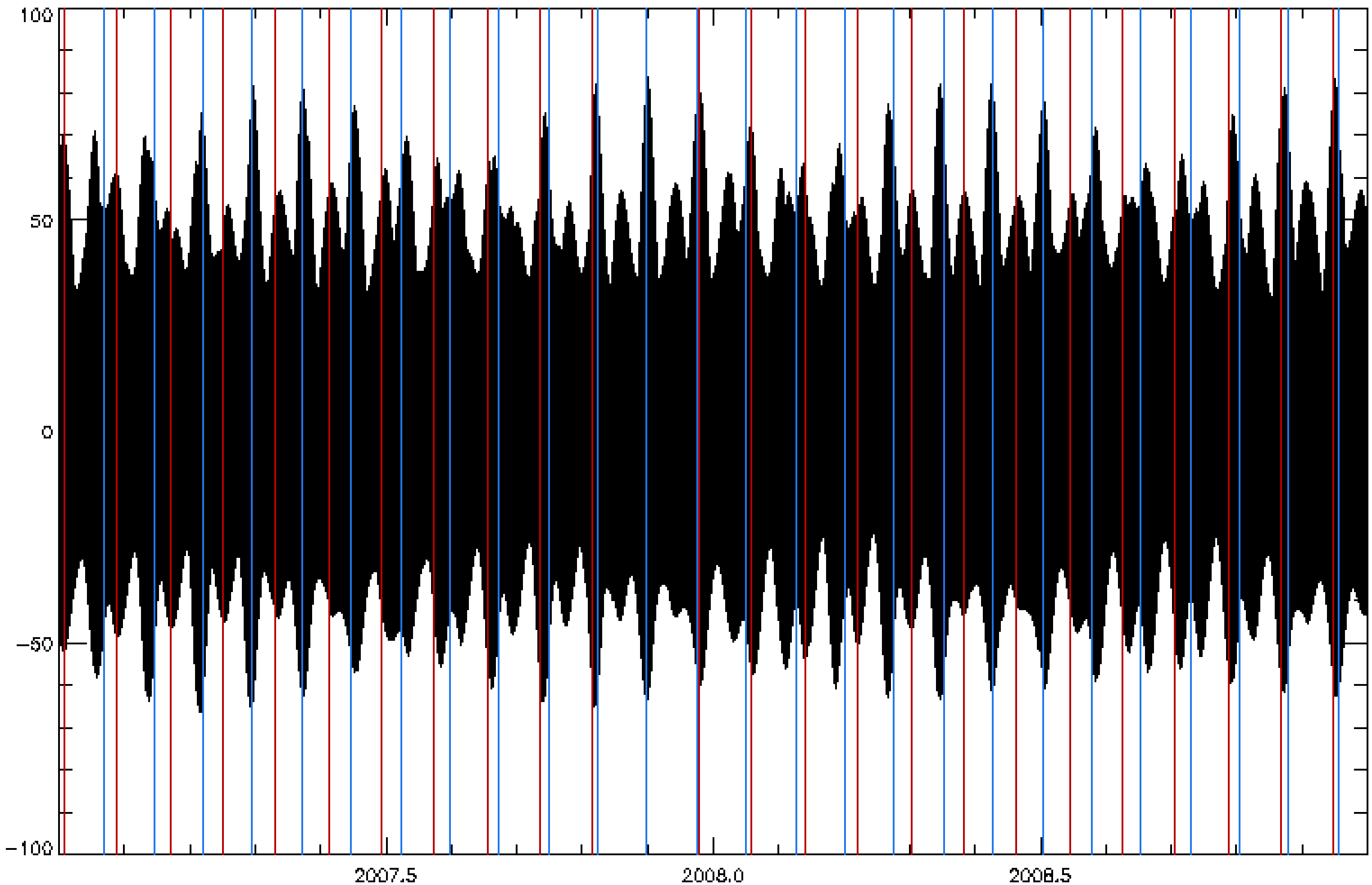
Natural scales much smaller than an ocean basin, strongly meandering currents and eddies. Clear ring formation in places. The variability is dominated by instabilities of the flow, rather than changes in the atmospheric forcing – all completely nonlinear.

Ocean bottom pressure in mbar, at 1100 m depth on the continental slope near Boston (1 mbar is equivalent to about 1 cm of sea level)



Spring-neap cycle (largest tides at full and new Moon)
Diurnal as well as semidiurnal (alternate tides of different amplitude)

The tide at a spot off the coast near Boston



Full Moon

Lunar perigee

All beautifully regular, predictable, and linear: higher tides when the sun and moon line up, or when the moon is closer. Highest tides when both occur simultaneously. Extremely linear behaviour coexisting with the nonlinear ocean currents.

What determines whether things are linear or nonlinear?

- **Wave speed**: if the waves travel much faster than the currents, then the currents can be ignored and this source of nonlinearity is not important.
- The waves responsible for tides (barotropic Kelvin waves) travel at around 200 m/s, much faster than the currents (0.01 to 1 m/s).
- The waves associated with open ocean currents (baroclinic Rossby waves) travel much more slowly (less than 0.1 m/s outside the tropics).
- **Wave amplitude**: if particle displacements by the wave are sufficient to change the wave speed significantly, the waves become nonlinear (for tides, and waves at the beach, the wave peak overtakes the trough, and waves break – tides do become nonlinear when the water is shallow enough, even becoming tidal bores in places).

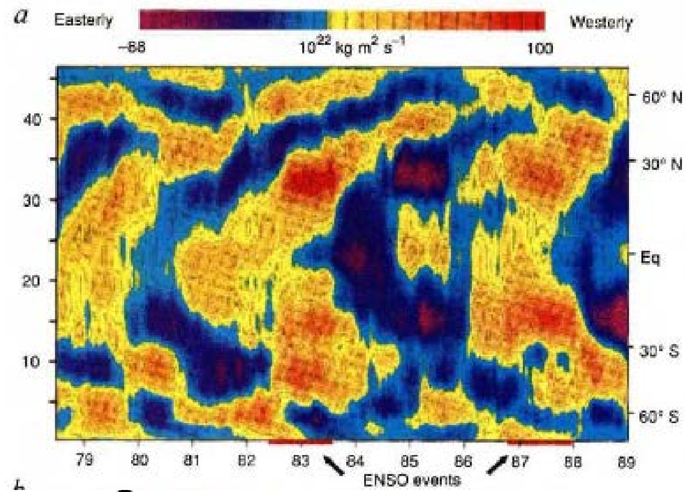
Linear and nonlinear behaviour coexist for aspects of the system controlled by different kinds of wave.

Global propagation of interannual fluctuations in atmospheric angular momentum

1992, 72 citations

J. O. Dickey*, S. L. Marcus* & R. Hide†

* Space Geodetic Science and Applications Group,
Jet Propulsion Laboratory, California Institute of Technology,
4800 Oak Grove Drive, Pasadena, California 91109, USA
† Meteorological Office Unit, Robert Hooke Institute, The Observatory,
Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK



Earth's Variable Rotation

1990, 154 citations

Raymond Hide and Jean O. Dickey*

Proc. R. Soc. Lond. A **387**, 31–73 (1983)
Printed in Great Britain

1983, 358 citations

Atmospheric angular momentum fluctuations, length-of-day changes and polar motion

By R. T. H. Barnes, R. Hide, F.R.S., A. A. White and C. A. Wilson
Geophysical Fluid Dynamics Laboratory, Meteorological Office (Met O 21),

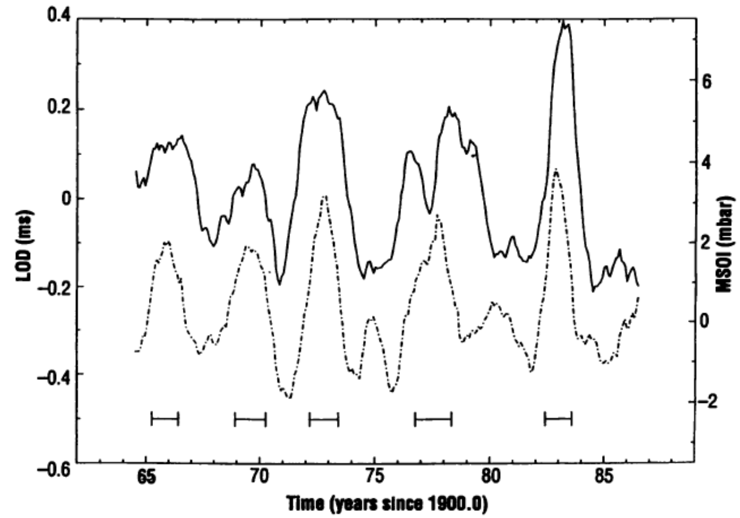


Fig. 9. The interannual LOD variation (upper curve) Λ_B and the MSOI (lower curve) from 1964 through 1986. The MSOI is defined as the negative of the 1-year moving average of the Southern Oscillation Index. ENSO events are denoted by the bars at the bottom (*19*).

1980, 110 citations

Atmospheric angular momentum fluctuations and changes in the length of the day

R. Hide*, N. T. Birch*, L. V. Morrison†, D. J. Shea‡ & A. A. White*

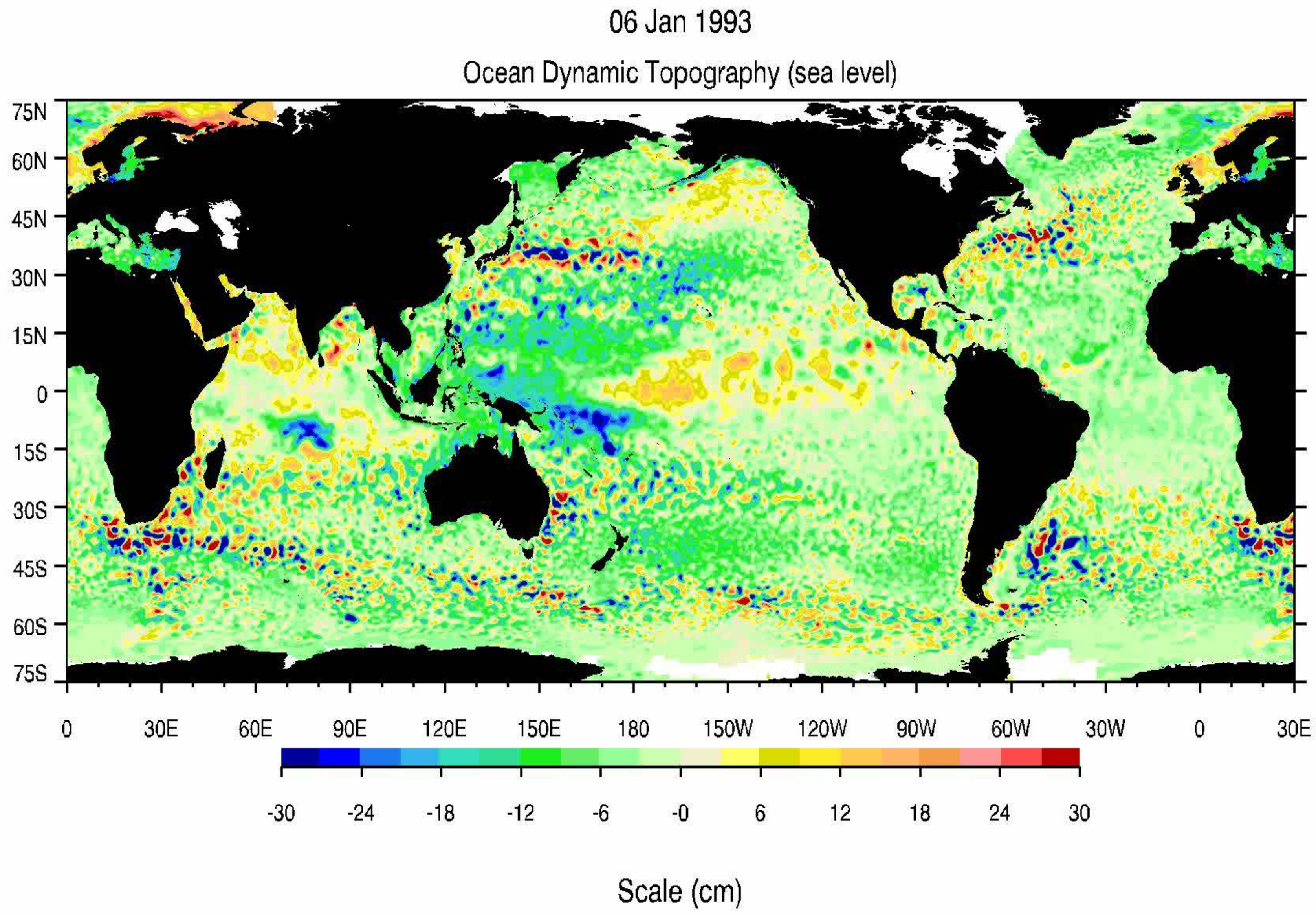
*Meteorological Office, Bracknell, Berkshire RG12 2SZ, UK
†Royal Greenwich Observatory, Herstmonceux Castle, Hailsham, East Sussex BN27 1RP, UK
‡National Center for Atmospheric Research, Boulder, Colorado 80307

Rossby waves operate like gyroscopes.

The link between spin direction and mass (geostrophic balance) means **they always go west** (unless the water goes faster to the east).

They go faster near the equator and slower near the poles, for the same reason as a gyroscope.

The speed of Rossby waves is determined by latitude, and the ratio of spin to mass.


06 Jan 1993
Ocean Dynamic Topography (sea level)
75N
60N
45N
30N
15N
0
15S
30S
45S
60S
75S
0
30E
60E
90E
120E
150E
180
150W
120W
90W
60W
30W
0
30E
-30
-24
-18
-12
-6
-0
6
12
18
24
30
Scale (cm)

Using the analogy with gyroscopes, the propagation speed is

$$c = -\frac{\beta}{2f}\frac{S}{M}$$

where S is the spin angular momentum and M is the mass anomaly of any isolated, propagating disturbance (single active layer).

For small interface perturbations and quasigeostrophic flow: $$\frac{S}{M} = \frac{2g'H}{f}$$

resulting in the long Rossby wave speed.

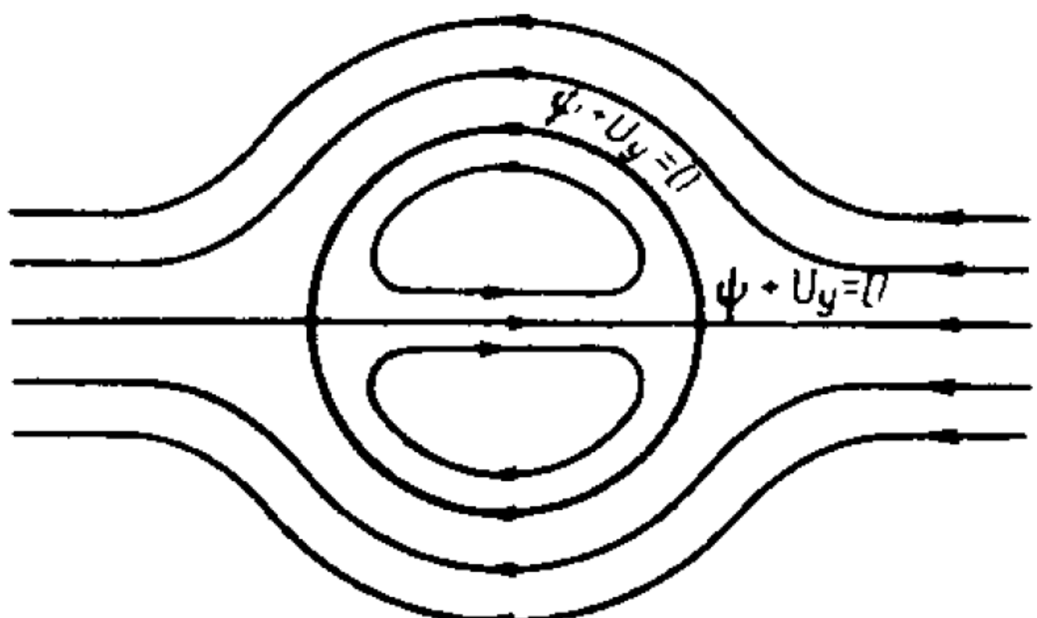
$$c = -\frac{\beta g'H}{f^2}$$

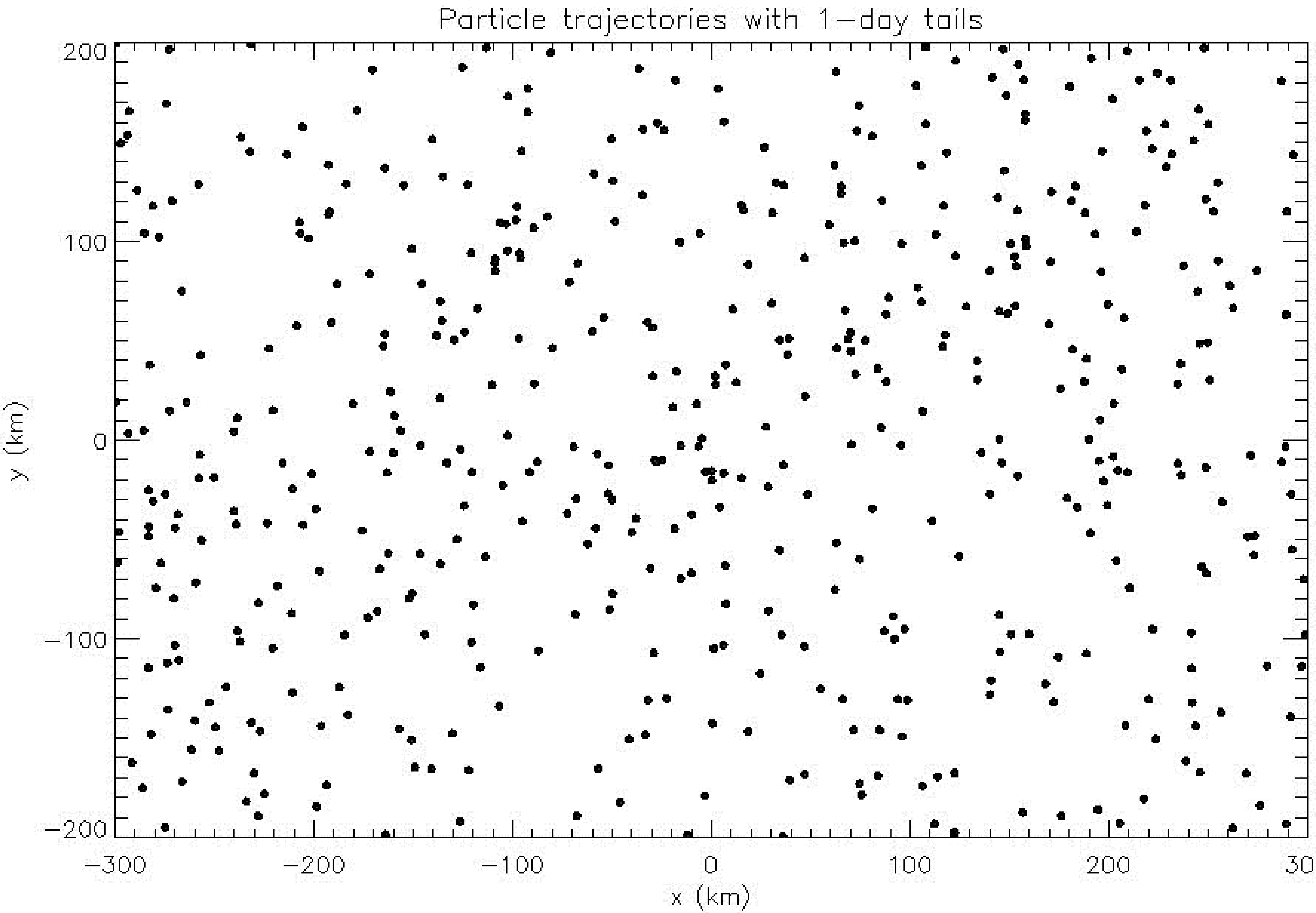
But what if S and M are close to zero?

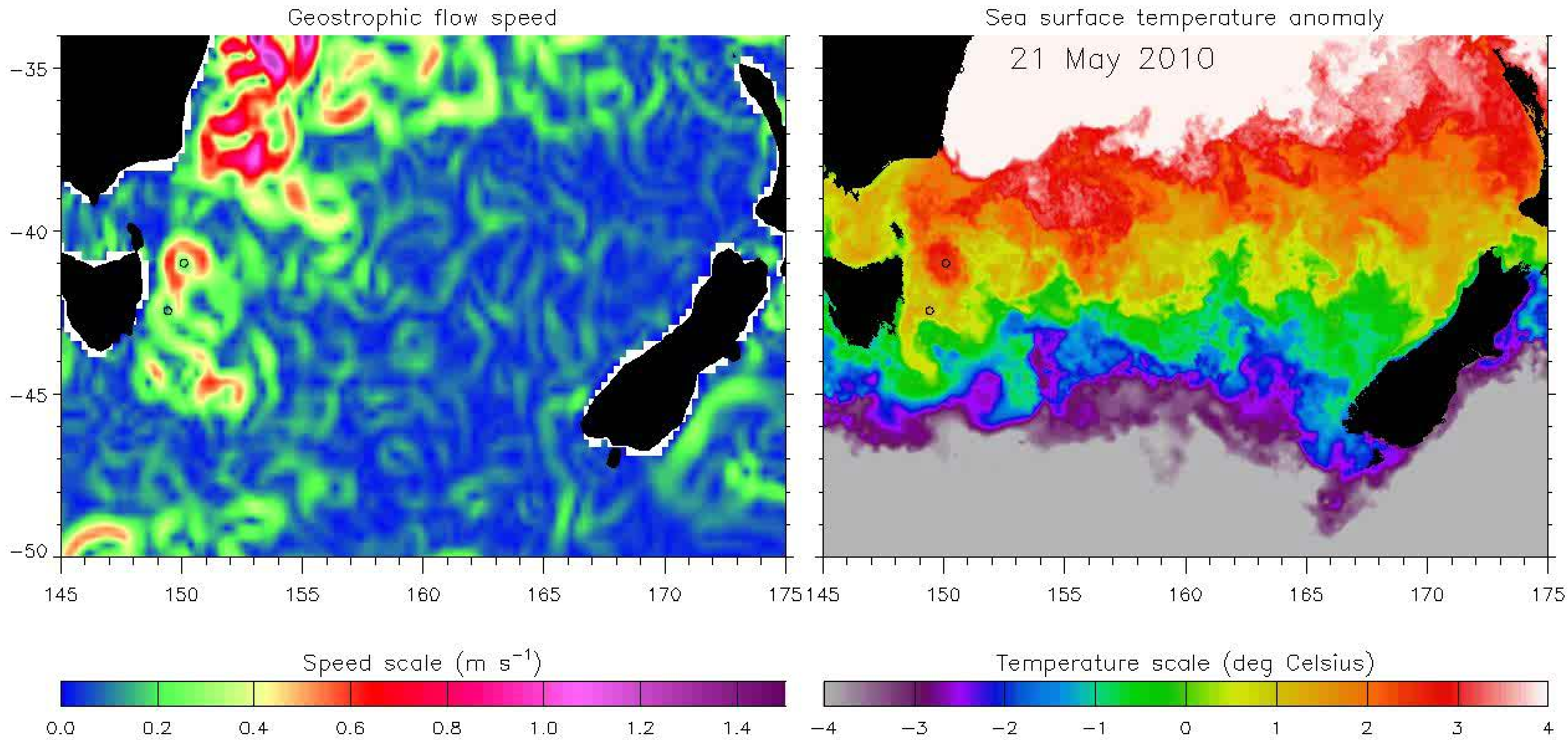
- If S and M are both close to zero, the small ageostrophic terms become dominant.
- Melvin Stern (1975) first noted this, and proposed a structure which could be stationary, which he named a “modon”
- Larichev and Reznik (1976) extended this idea to other speeds outside the range of linear Rossby waves.

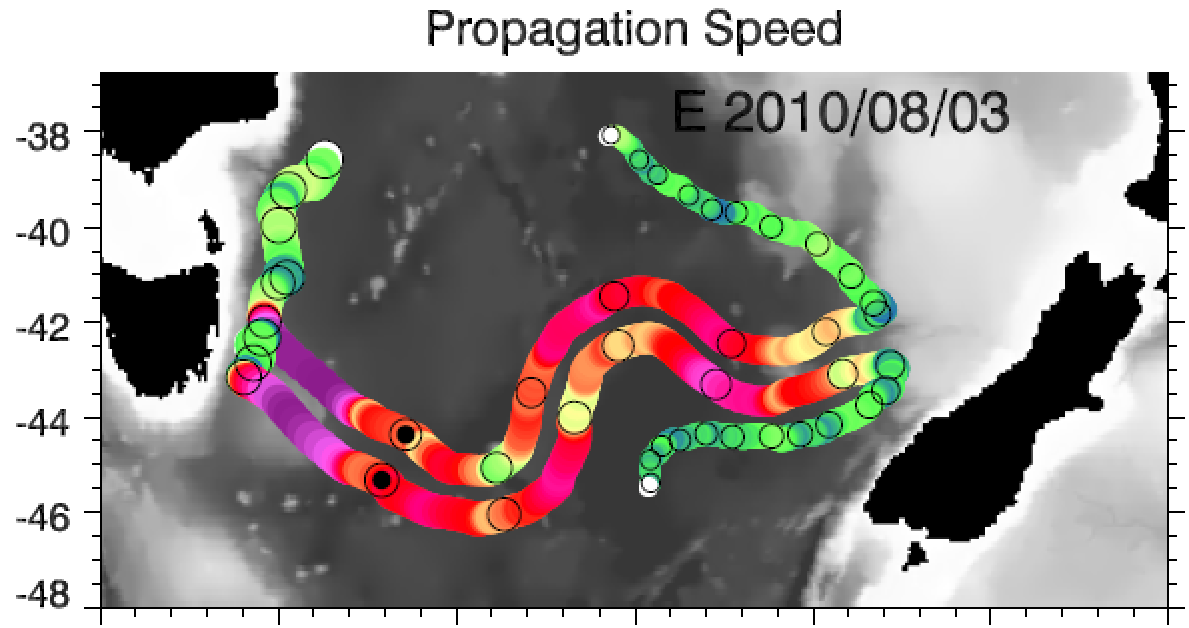
TWO-DIMENSIONAL SOLITARY ROSSBY WAVES[1]

V. D. Larichev and G. M. Reznik

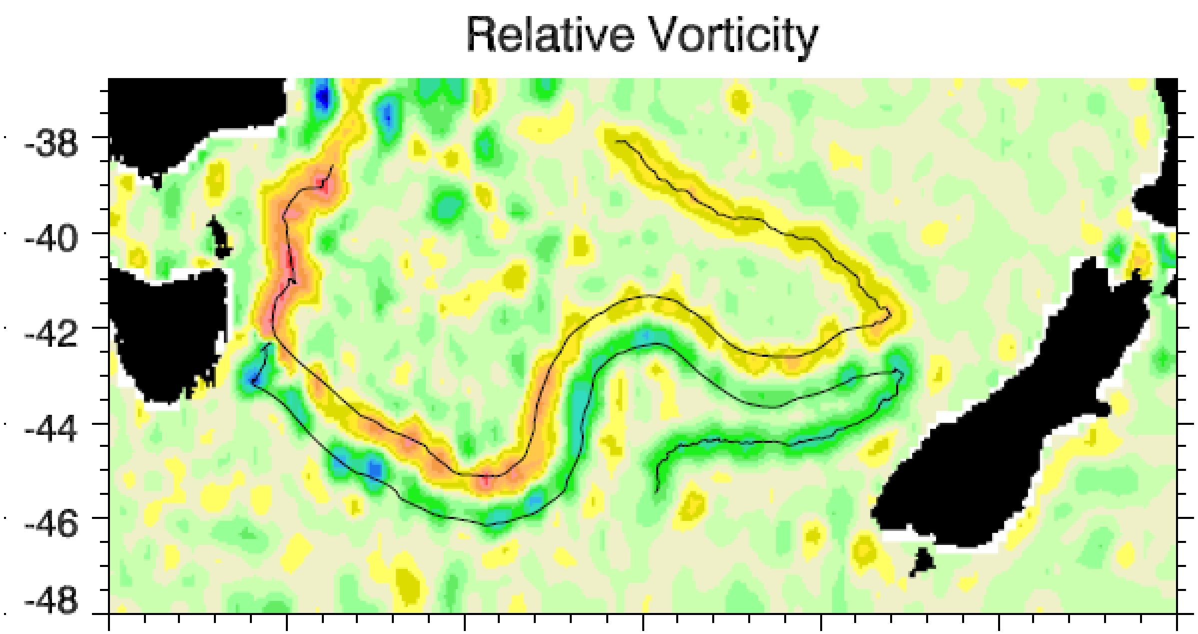


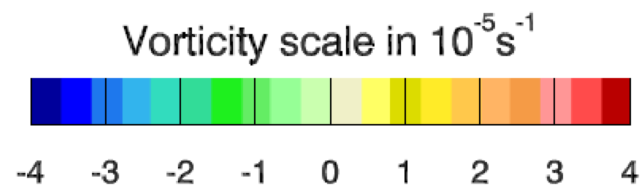

Particle trajectories with 1-day tails
200
100
0
−100
−200
y (km)
−300
−200
−100
0
100
200
300
x (km)


Geostrophic flow speed
Sea surface temperature anomaly
21 May 2010
-35
-40
-45
-50
145
150
155
160
165
170
175
Speed scale (m s-1)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Temperature scale (deg Celsius)
-4
-3
-2
-1
0
1
2
3
4

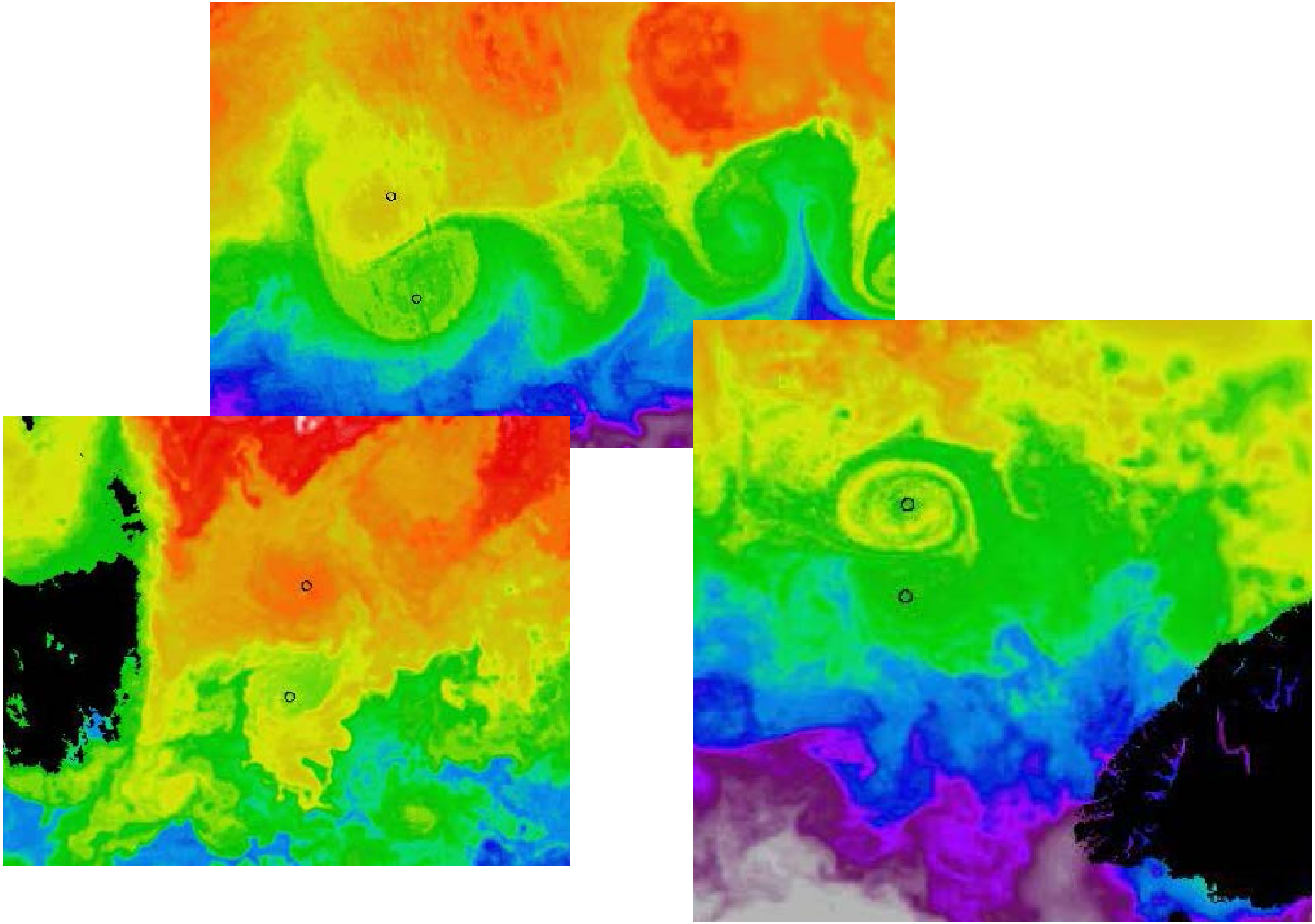


Linear long Rossby wave speed in this region: 1.2 cm/s





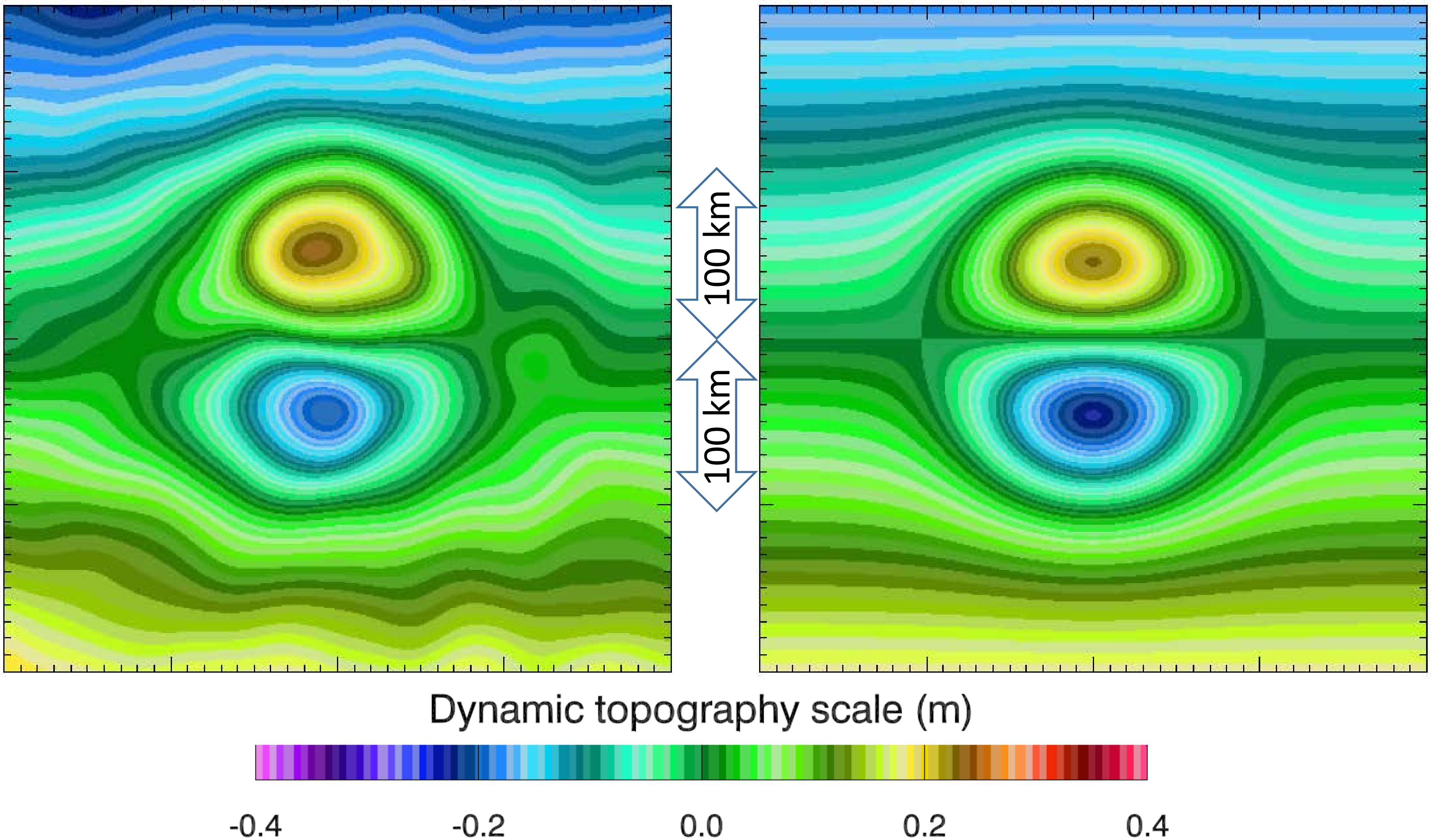
Black circles every 30 days, black dot on the date shown.



Simultaneous SST images showing fine scale structure

Observed when averaged in moving and twisting reference frame

Predicted structure (Larichev+Reznik, 1976)

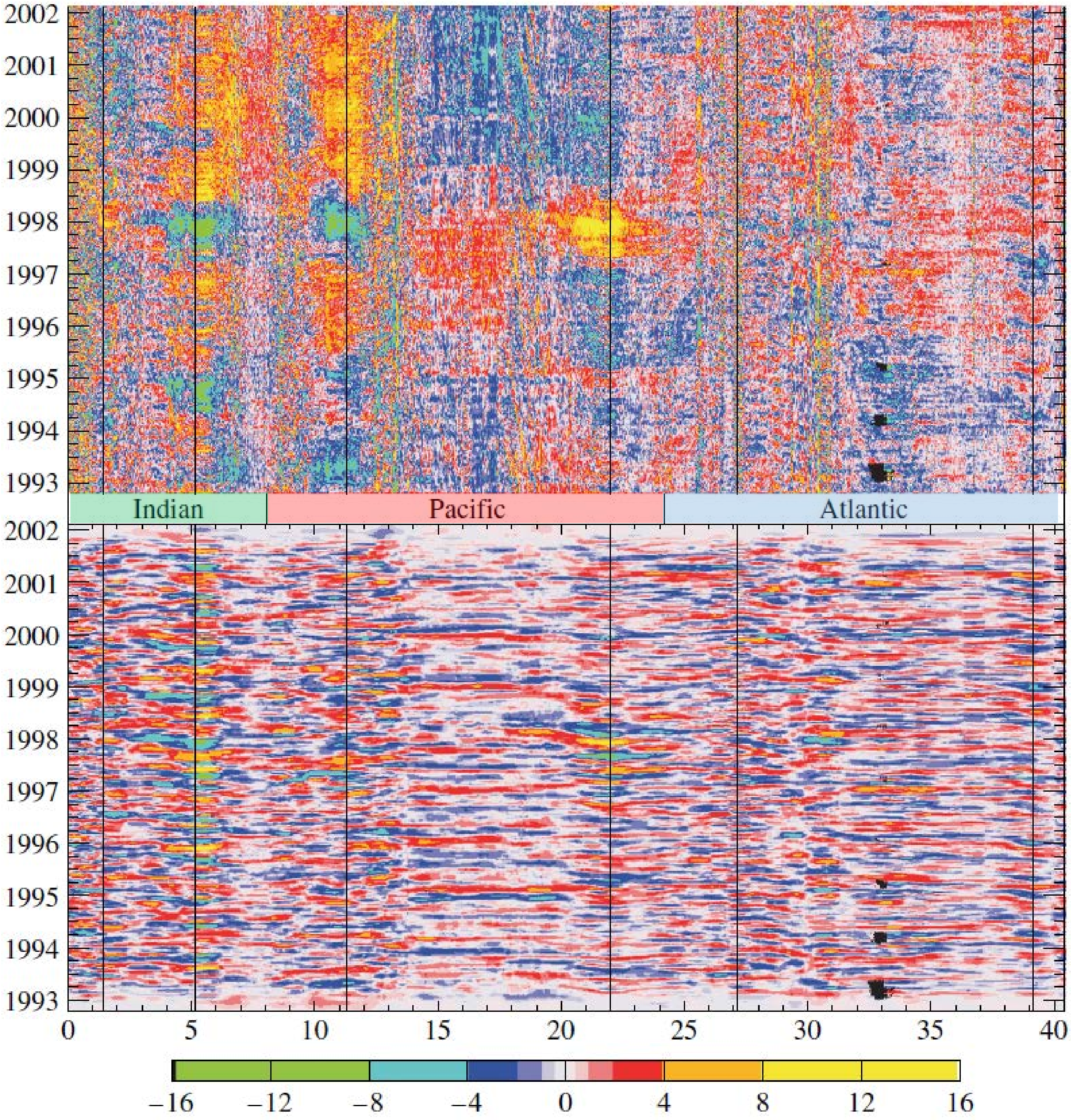


Prediction requires as inputs: Rossby radius (27 km, from hydrography), propagation speed (average = 8.8 cm/s over this period), latitude, and radius (best-fitting value = 100 km). Spatial structure and amplitude are both outputs of the calculation.

But what about boundaries

- The ocean has walls. What happens when a gyroscope hits a wall?
- Boundaries are different, and the associated wave speeds are different too – much faster than Rossby waves.
- Waves propagate much more quickly than Rossby waves, because the walls supply the torque needed.
- Boundary waves of several kinds exist, but all propagate with land on their right (N Hemisphere), or left (S Hemisphere).
- Have to look carefully at the data to see what happens at the boundaries.

black, vertical lines mark the equator

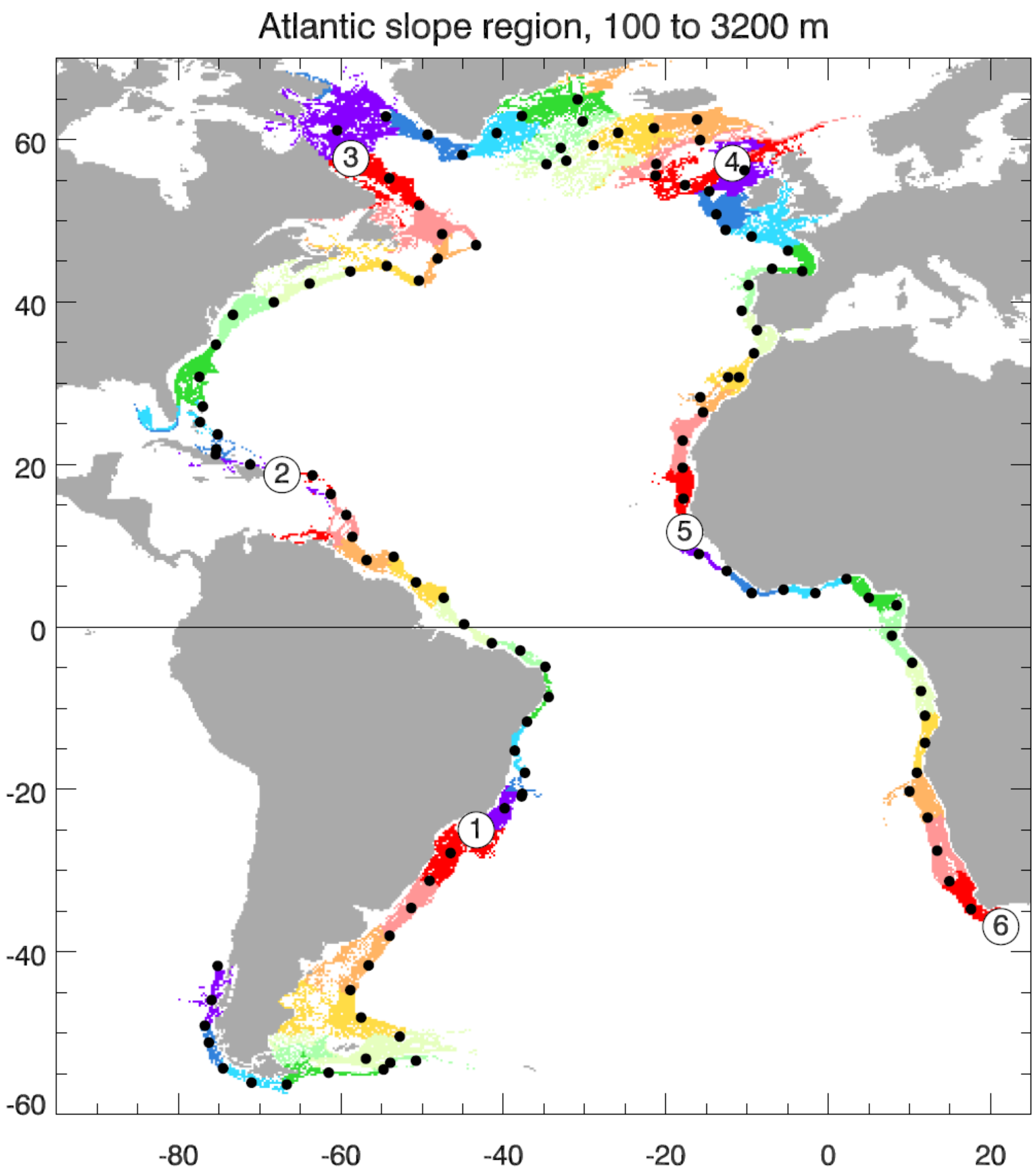


Sea level (cm) anomalies as a function of time, and distance **along the global continental slope.**

Unfiltered (use double the scale below)

Filtered: periods < 1 year, and 100 grid point average along the slope (1 tick mark).

Correlations visible over very long distances.



1/12 degree global ocean model (sufficient resolution to have points on the continental slope – climate models can have one at the top and one at the bottom!

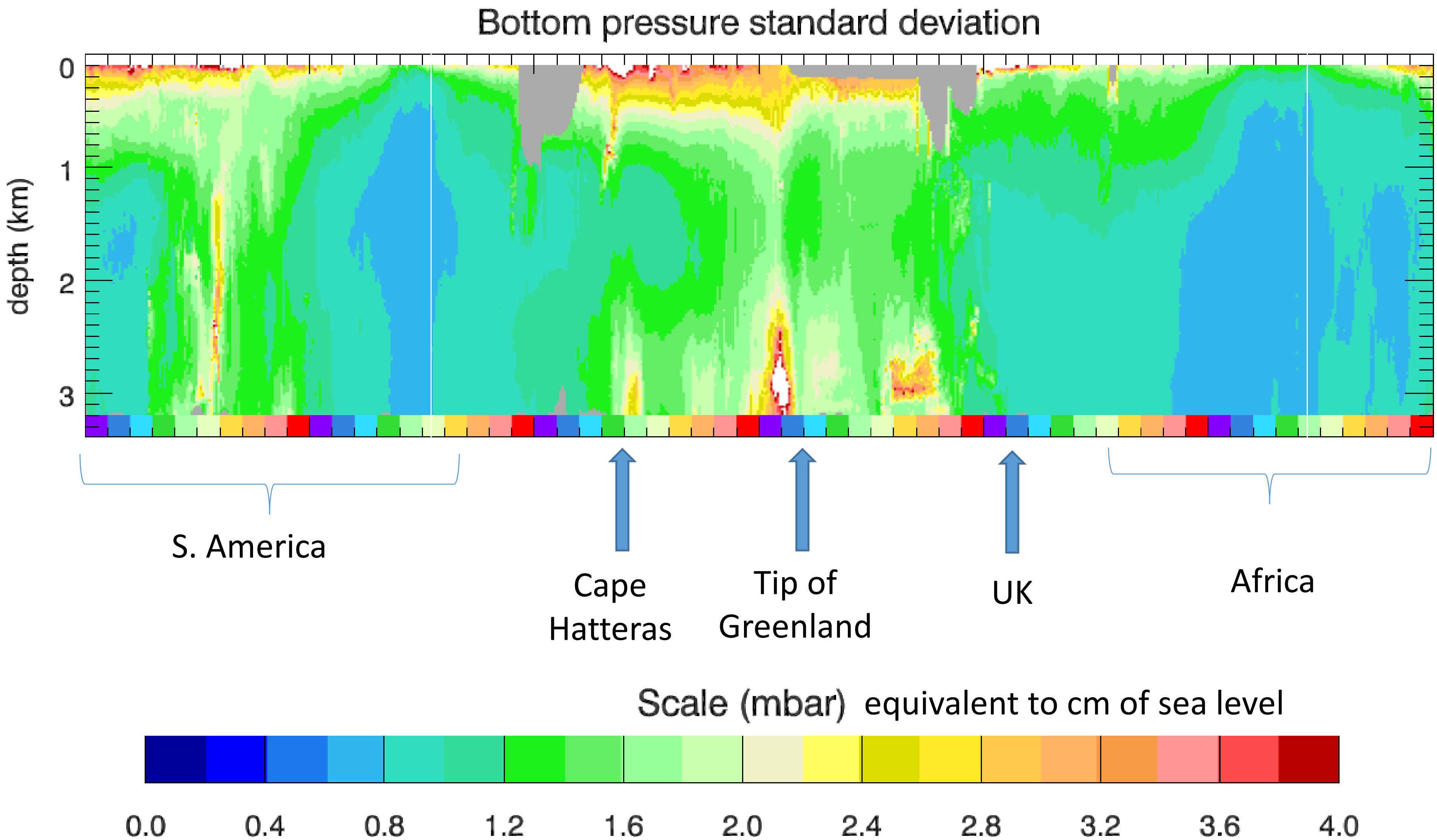
Extract bottom pressure data from the continuous contours on the Atlantic continental slope.

Depths 100 – 3200 m.

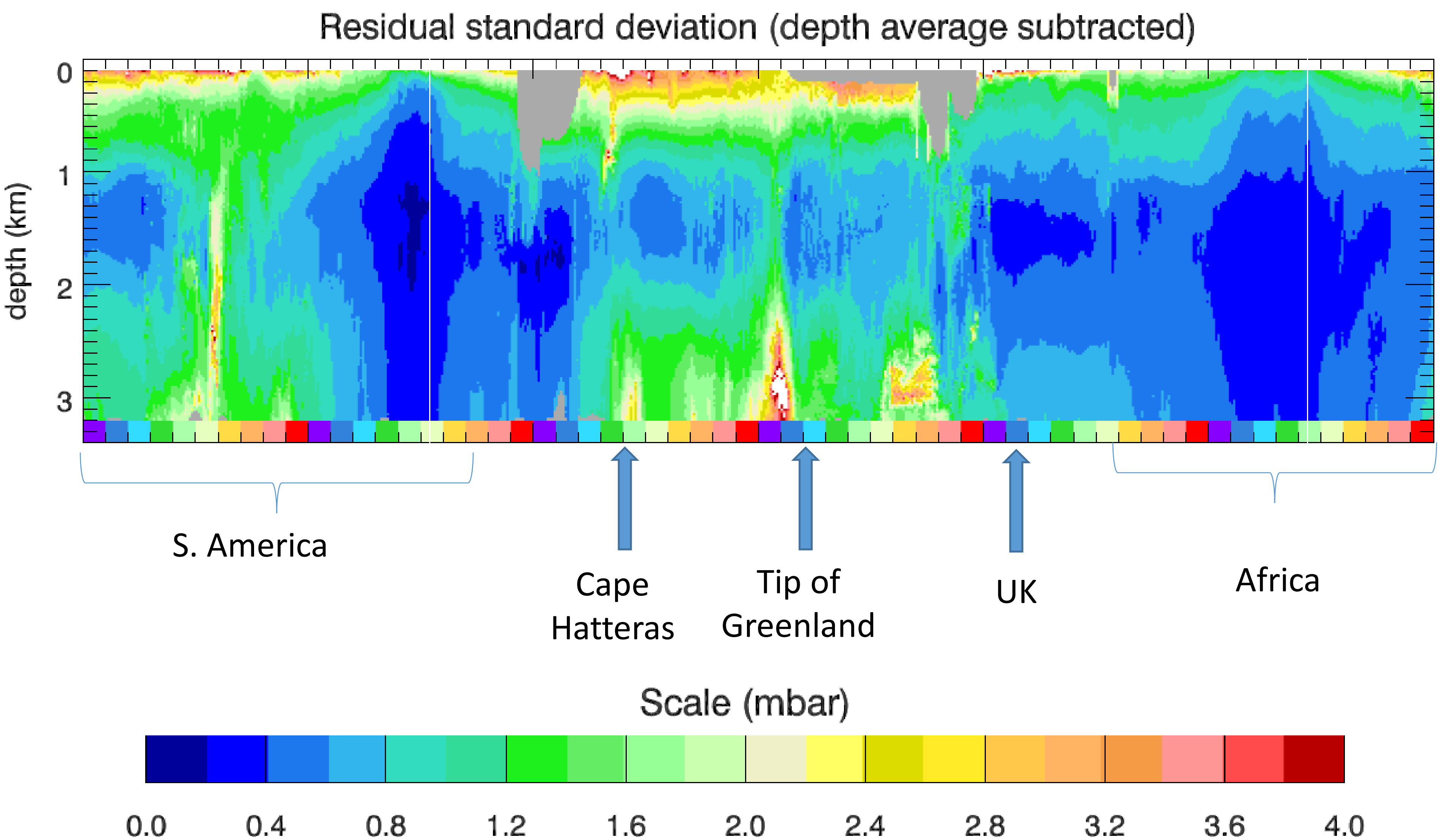
Plot diagnostics as a function of depth and distance along the slope.

Numbers represent distances in units of 10,000 km. Dots are every 500 km, colour changes every 1000 km.

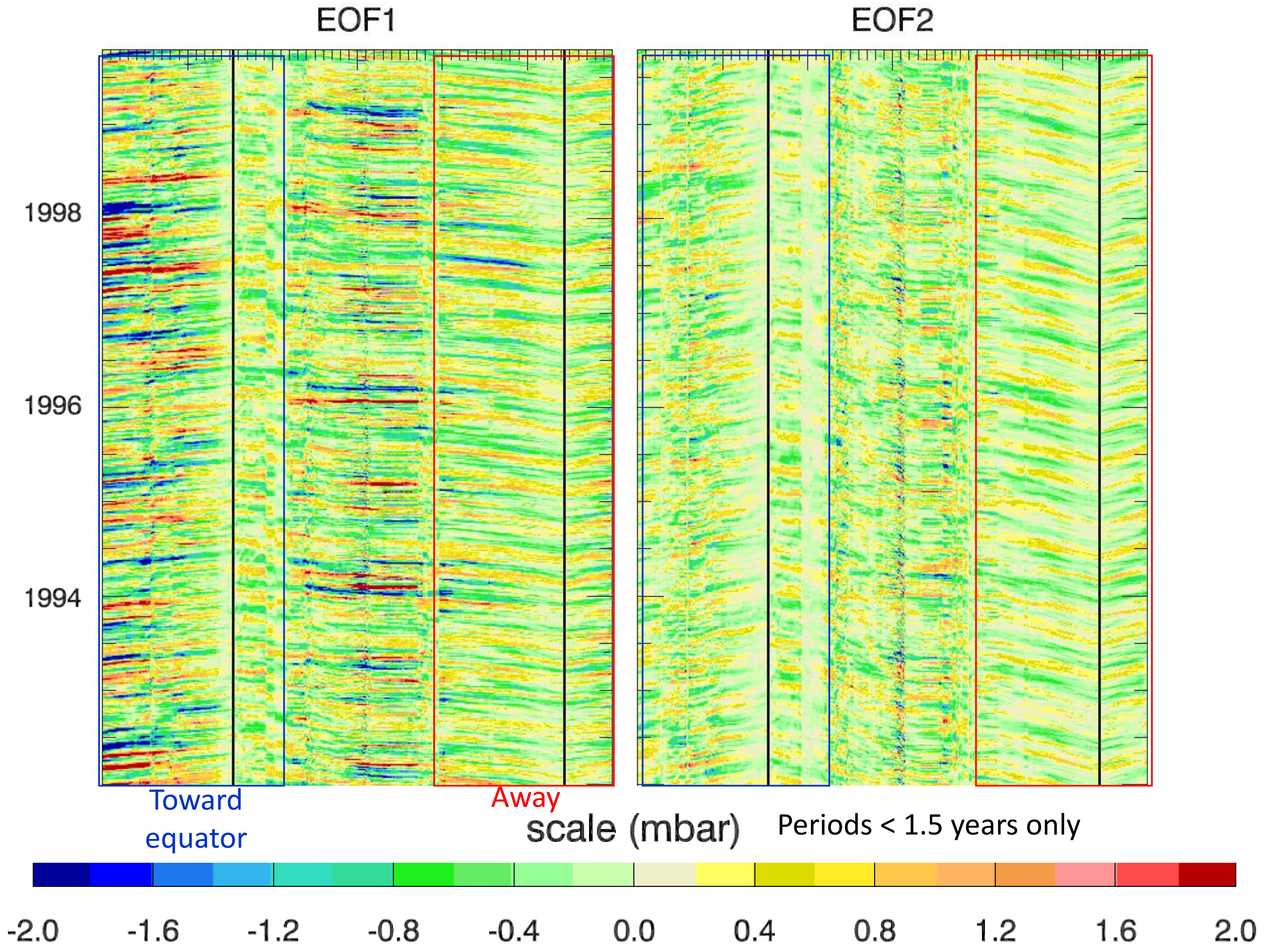
Standard deviation of model bottom pressure (**50 years of 5-day means**)
on the Atlantic continental slope
[looking into the Atlantic from the south, with the "walls" unwrapped]

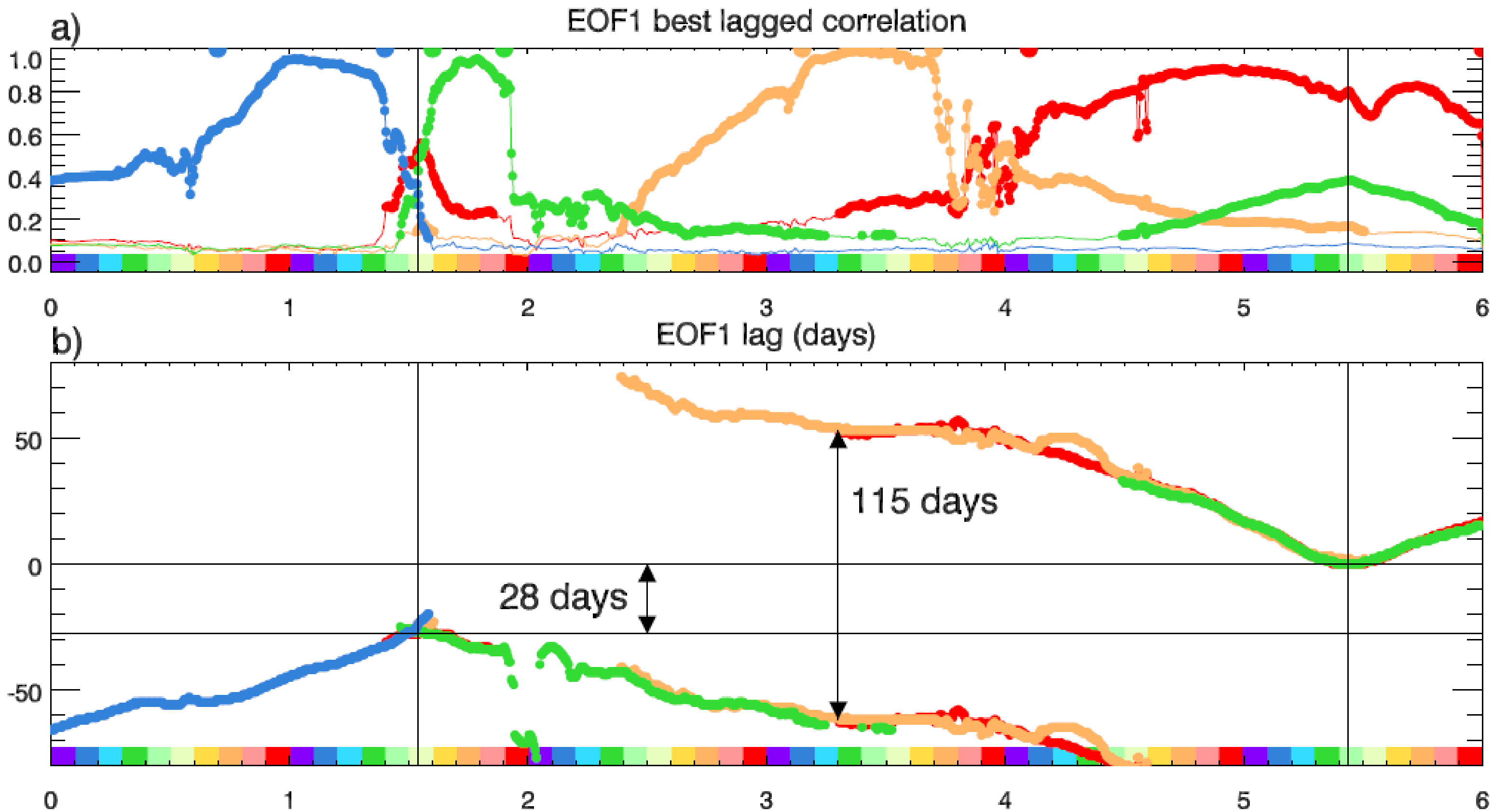


A large fraction of this continental slope variability is large scale barotropic



Less than 1 cm sea level equivalent over much of the domain


EOF1
EOF2
1998
1996
1994
Toward equator
Away
scale (mbar)
Periods < 1.5 years only
-2.0
-1.6
-1.2
-0.8
-0.4
0.0
0.4
0.8
1.2
1.6
2.0

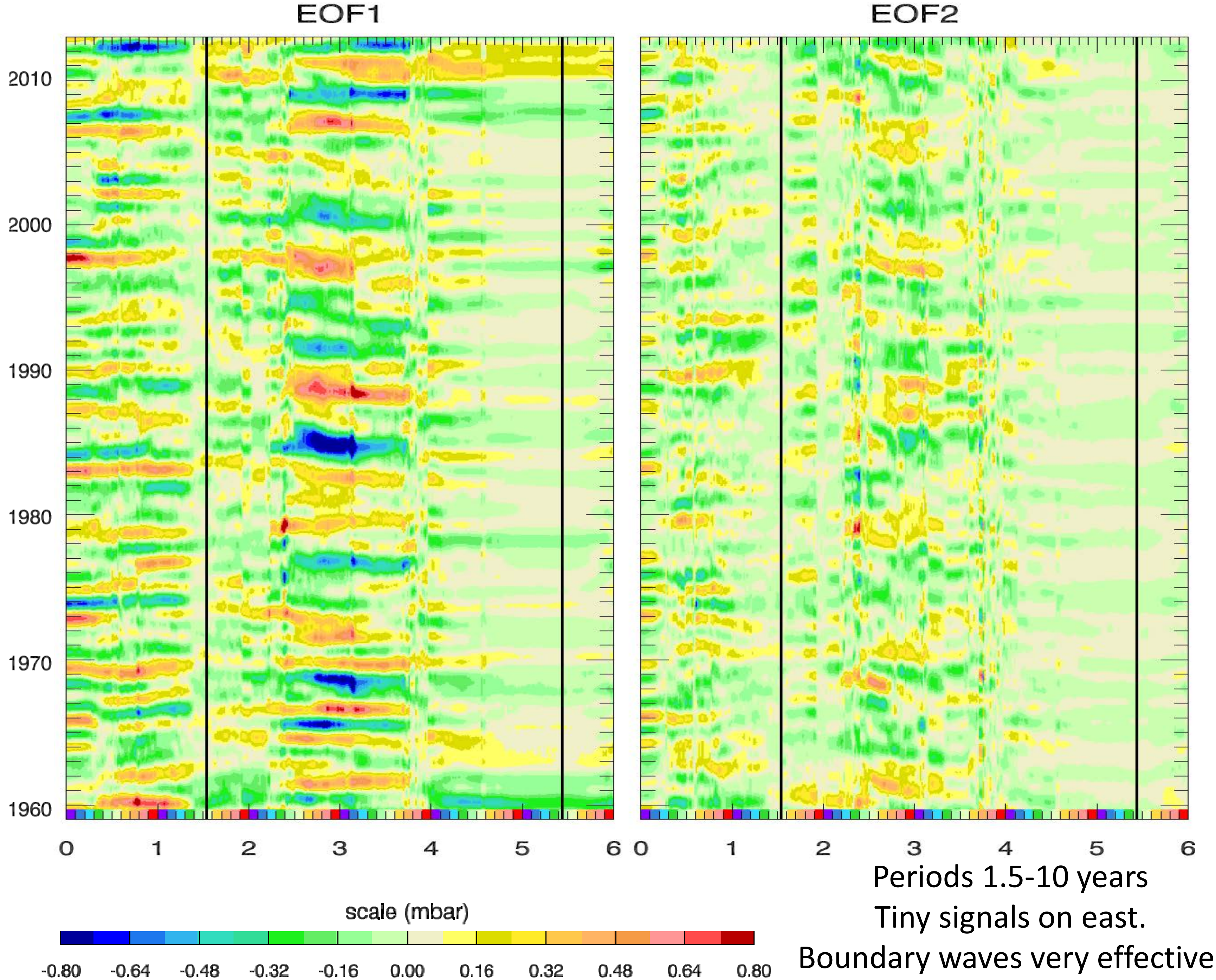


A circuit of the North Atlantic takes 115 days

The Equatorial Kelvin Wave takes 28 days (2.44 m/s)

The Boundary Wave takes 115 – 28 = 87 days (**3.7 to 5.2 m/s** depending on choice of length)

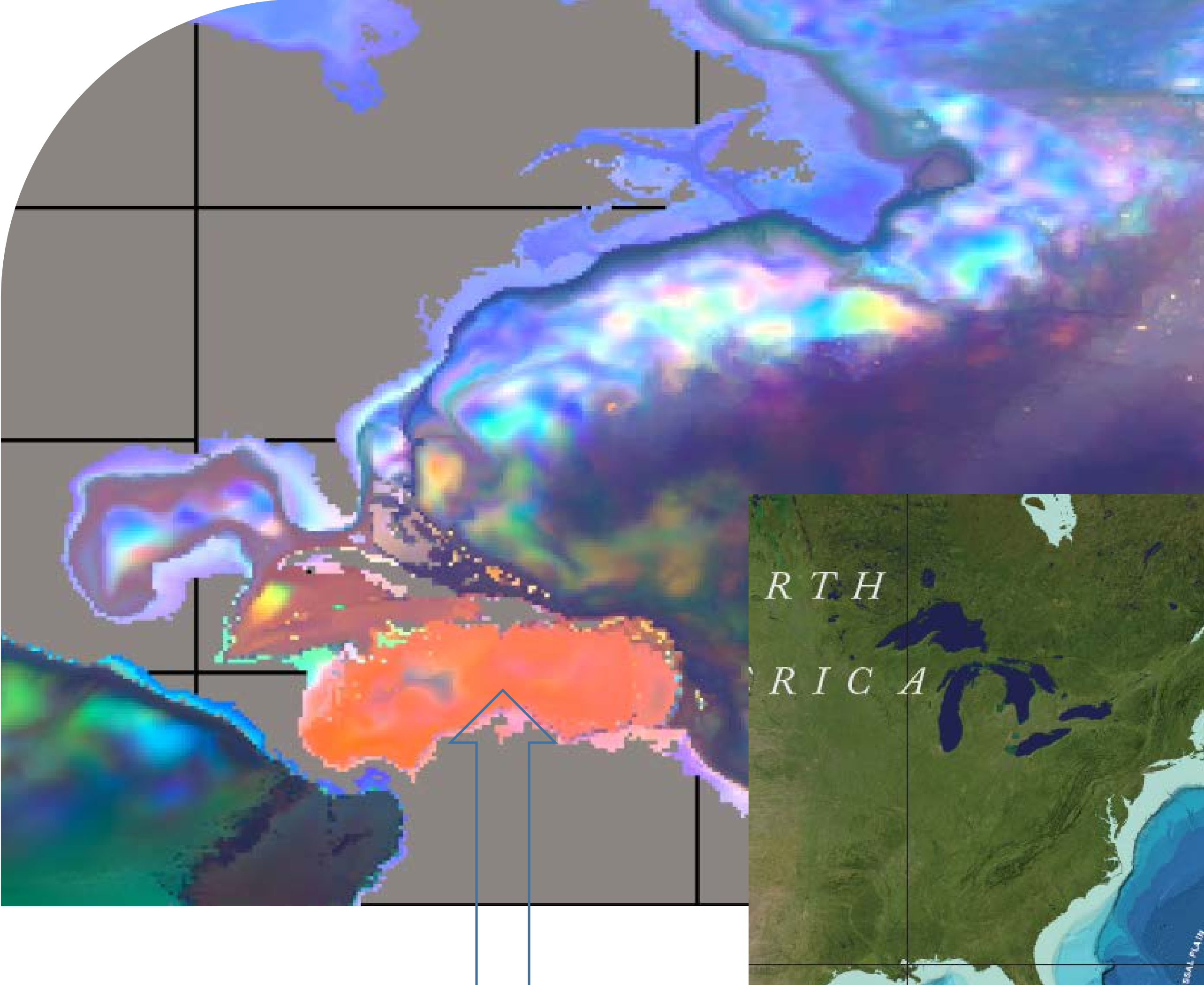
Not as fast as tides, but still significantly faster than ocean currents.



Periods 1.5-10 years
Tiny signals on east.
Boundary waves very effective.

Some consequences

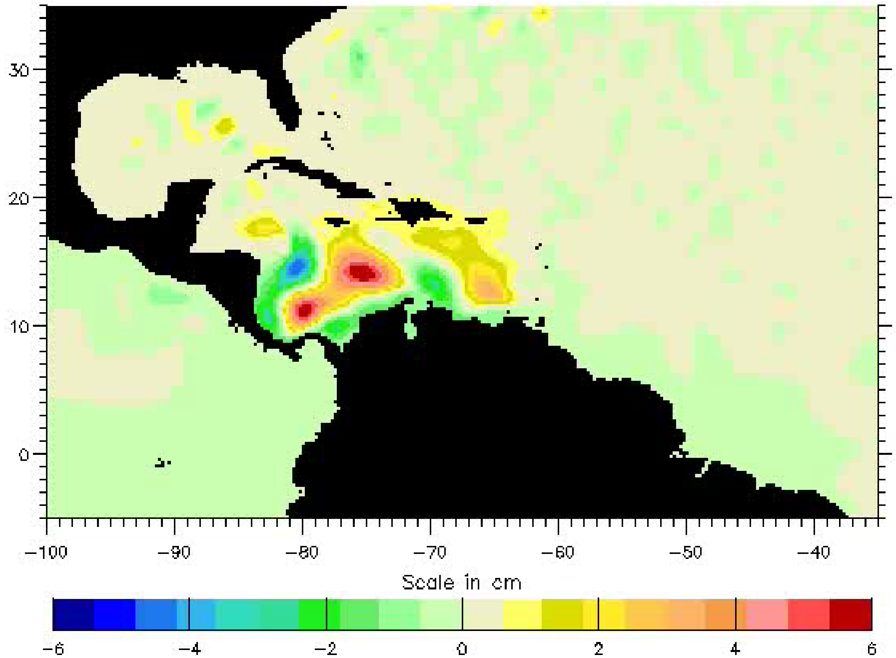
- Eastern boundary pressures really are a “fixed point” for the ocean circulation.
- We can monitor global scale ocean dynamics with relatively few measurements, as long as they are on the continental slope (and these boundary pressures tell us the integrated northward flow, and hence the Meridional Overturning Circulation).
- Rossby wave + boundary wave -> basin modes: The Rossby Whistle.



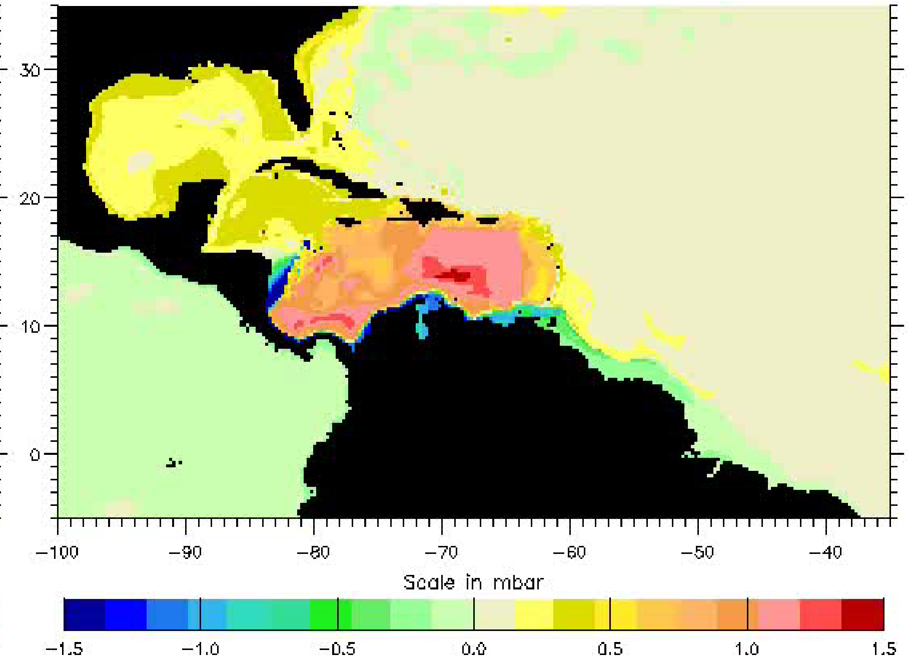
Bottom pressure variability
brightness = amplitude
colour represents spectrum shape

What is so special about here?

Sea Level



Bottom Pressure



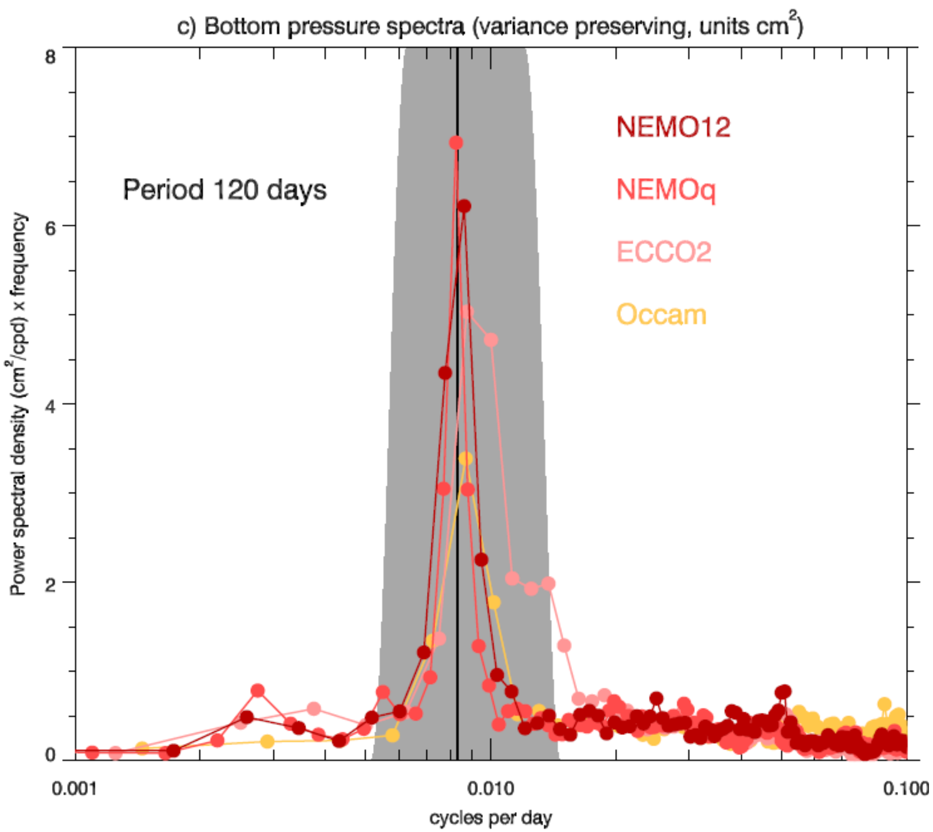
A 120-day period basin mode consisting of a Rossby wave converting to boundary wave and back again.

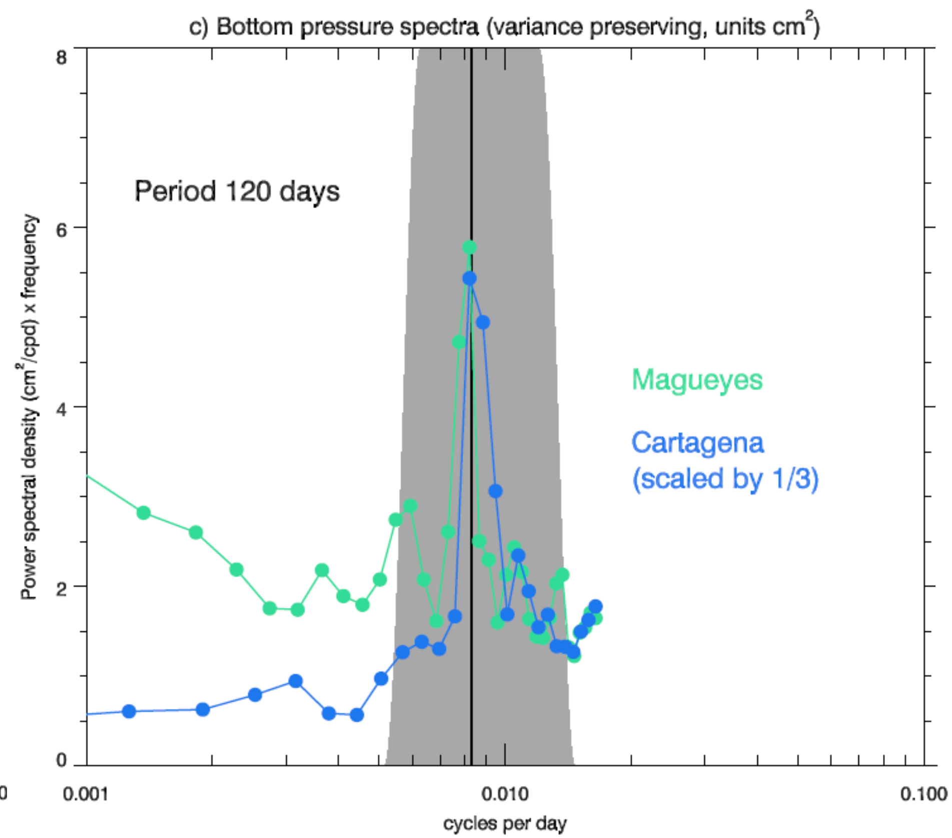
The mode is a resonant, linear mode, but its energy comes from instability of the Caribbean Current which flow to the west through the basin.

This is analogous to how a whistle works – an instability of the mean flow energising a linear normal mode of the cavity.

Hughes et al., 2016:A Rossby whistle:
A resonant basin mode observed in the Caribbean Sea. *Geophys. Res. Lett*. **43**, 7036-7043, doi: 10.1002/2016GL069573

Very sharply peaked spectrum at period 120 days



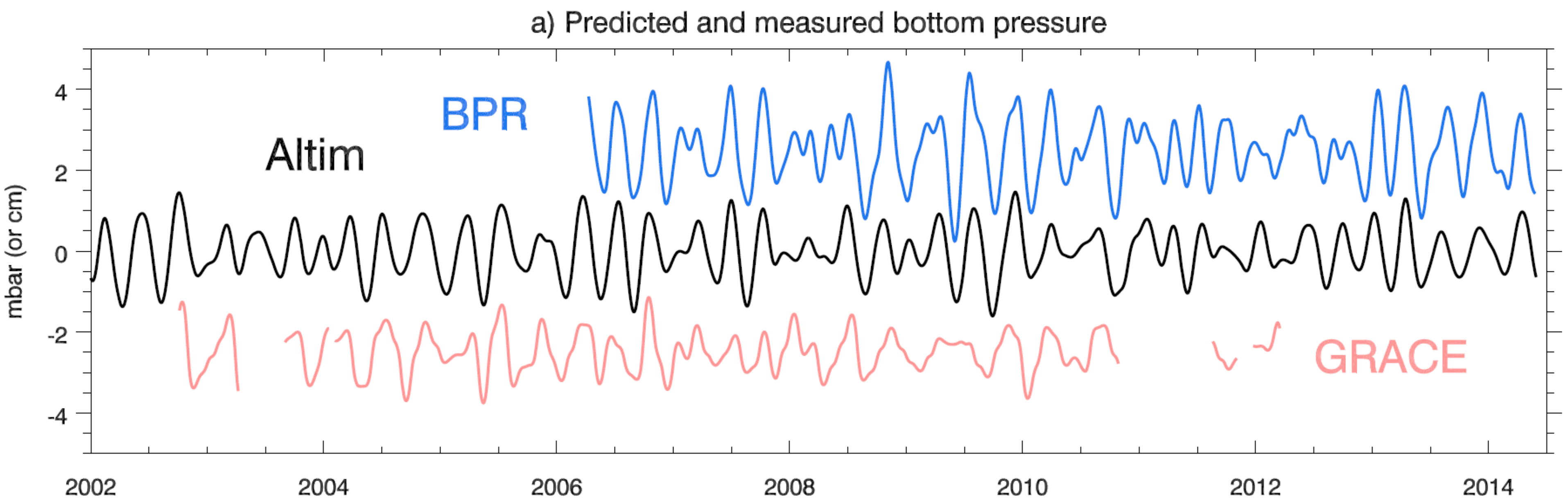


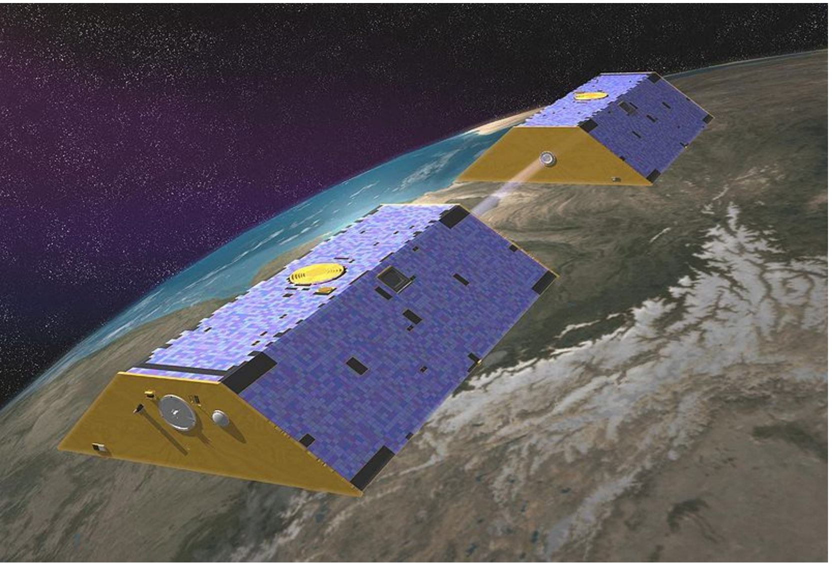
Model bottom pressures

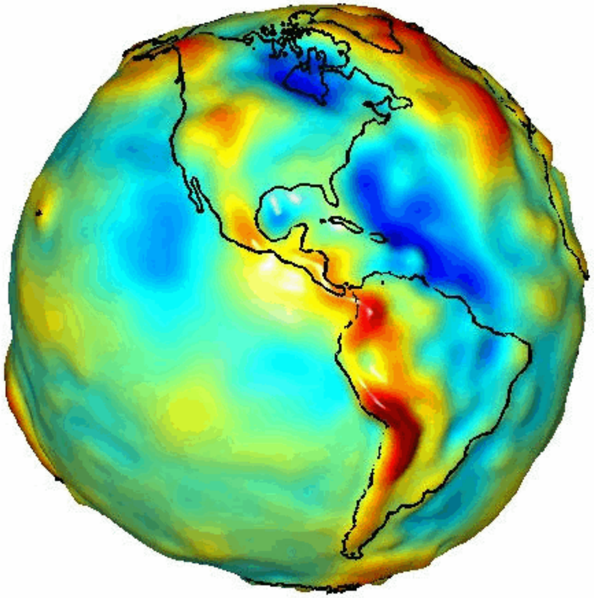
Caribbean Sea tide gauges

The "mesoscale" can have an effect on coastal sea level in this resonant case

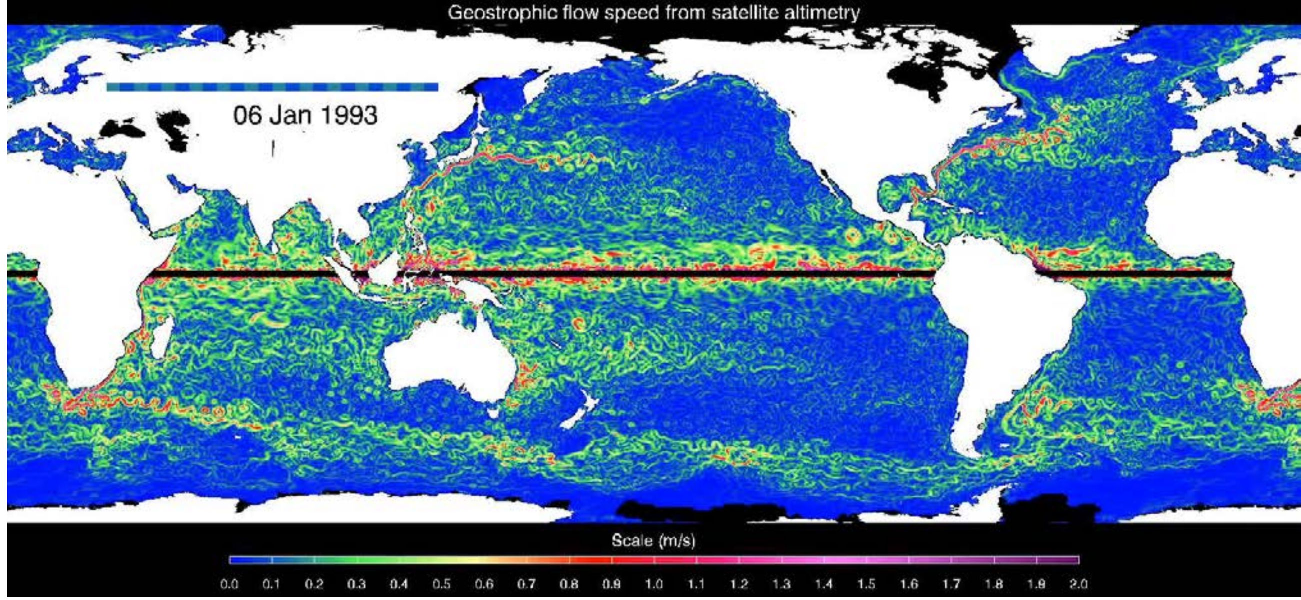
GRACE infers ocean bottom pressure from changes to the orbits of the satellites, because of the gravity anomalies associated with the mass changes







Conclusions



- The ocean has a mix of linear and nonlinear dynamics
- Consideration of wave speeds and angular momentum conservation helps understand how and why these regimes coexist
- The "Earth as a planet" view has enabled some very nice satellite measurements of the ocean
- There is a menagerie of behaviours in the ocean that we are beginning to get to grips with

- A big thank you to Raymond for helping to point me in the right direction