

Problem Sheet

Question 1:

What follows is a list of some (not all!) terms and concepts commonly used in space plasma physics, which you will often hear in group talks, for example. For each term, make your own notes about what each term means or represents. Provide an equation or diagram as appropriate.

- Alfvén waves
- Cyclotron frequency
- Debye shielding
- 'E x B' drift
- Electron plasma frequency
- Friedrichs diagram and propagation anisotropy
- Frozen-in Fields
- Larmor radius
- Magnetic mirror
- Magnetic moment and adiabatic invariants
- Magnetic reconnection
- Magnetohydrodynamics
- Magnetosonic waves, fast and slow
- Magnetic Reynolds number
- Pitch angle
- Plasma beta

Question 2:

Write down the definitions of ω_{pe} , λ_D , and $n\lambda_D^3$ and calculate their values for the following plasmas

	n (m^{-3})	$T_e(K)$
Solar corona	10^{16}	10^6
Solar wind	10^7	10^5
Ionosphere	10^{12}	10^3
Fusion reactor	10^{21}	10^8

Question 3:

Write down the MHD momentum equation in the absence of gravity. Assuming that the plasma is at rest, simplify the equation and write down an equation relating the gradient in the plasma pressure to the current density and the magnetic field.

If the plasma beta is small, then the magnetic field pressure dominates. Further simplify your equation in this limit and show that $\mathbf{j} \times \mathbf{B} = 0$.

If $\mathbf{j} \times \mathbf{B} = 0$ then \mathbf{j} is parallel to \mathbf{B} , e.g. $\mathbf{j} = \alpha \mathbf{B}$. Show that such a magnetic field satisfies the Helmholtz equation, i.e.

$$(\alpha^2 + \nabla^2)\mathbf{B} = 0$$

Such conditions apply in the solar corona. The corona is very structured and exhibits loops of plasma which can exist in quasi-steady conditions for days. These loops are examples of flux tubes, or flux ropes.

One particular model of a flux rope is the Gold and Hoyle model which can be written in cylindrical polar coordinates as:

$$B_\theta(r) = B_0 \left(\frac{r}{a} \right) / \left(1 + \frac{r^2}{a^2} \right)$$

$$B_z(r) = B_0 / \left(1 + \frac{r^2}{a^2} \right)$$

Demonstrate that the model flux rope is force-free, i.e. $\mathbf{j} \times \mathbf{B} = 0$.

Sketch the configuration of the magnetic field lines in a flux rope.