Space Physics Problem Sheet 1

Question 1:

We have discussed in lectures the concept of the Debye length of a plasma and quasi-neutrality, where for space plasmas it can be assumed that they are quasi-neutral when examined on spatial scales larger than the Debye length, $\lambda_{D.}$

a)Show that from

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi(r)}{dr}\right)=\frac{-en_o}{\epsilon_o}\left[1-\exp\left(\frac{e\Phi(r)}{kT}\right)\right]$$

which we derived in lectures; we can show that for a sufficiently high plasma temperature, that:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi(r)}{dr}\right)\approx\frac{n_o e^2}{\epsilon_o kT}\Phi(r)$$

b) Demonstrate by substitution that the potential given by:

$$\Phi(r)=\frac{A}{r}\exp\left(\frac{-r}{\lambda_D}\right)$$

is a solution to the differential equation:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi(r)}{dr}\right)=\frac{1}{\lambda_D^2}\Phi(r)$$

c) Given that;

$$\epsilon_o=8.8542\times10^{-12}\,Fm^{-1};\quad k=1.3807\times10^{-23}\,JK^{-1};\quad e=1.6022\times10^{-19}C$$

show that a numerical expression for the Debye length is given by:

$$\lambda_D=69\sqrt{T/n_o}$$

where the units of Debye length are metres, T is in units of 10^6 K and n_o is in cm^{-3}, these being usual units in space physics for these quantities.

Let us now consider the value of λ_D for two typical space plasmas:

d) In the Earth's topside ionosphere typical values for the temperature and density are ~1000 K and 10^5 cm^{-3}. Calculate the Debye length of such a plasma and using the fact that the vertical and horizontal extents of the ionosphere are of the order of L ~ 300km and 3,000km respectively , decide whether such a plasma can be considered to be quasi-neutral.

e) Typical parameters in the solar wind near 1AU (~ 10^8km) are T ~ 10^5 K and n_o ~ 10 cm^{-3}. Calculate the solar wind Debye length and decide whether such a plasma is quasi-neutral.

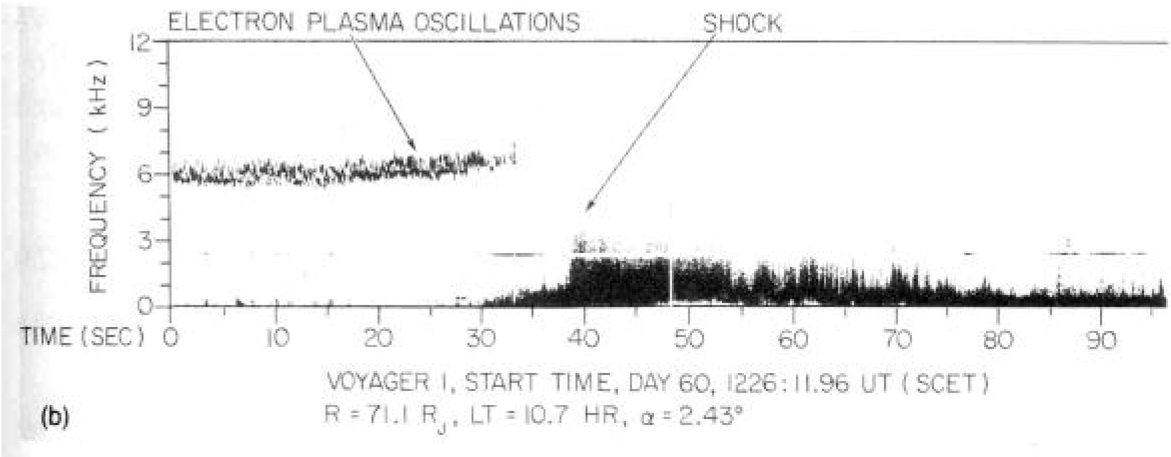
Question 2:

(a) We discussed in lectures how a spatially localised perturbation in a cold plasma will not propagate but will oscillate at the plasma frequency ω_{pe}. Using the values of the constants given in Question 1 and the fact that $m_e = 9.1095 \times 10^{-31}$ kg, show that this fundamental frequency can be written as

$$f_{pe} \approx 9 \times 10^3 \sqrt{n_o}$$

where the units of f_{pe} are Hz, and n_o are cm^{-3}.

(b) An example of electron plasma oscillations observed upstream of the bow shock of Jupiter is shown in the figure below. From the value of the frequency shown calculate what the solar wind density must have been.



c) Show that the Debye length and the electron plasma frequency can be related by the following relationship:

$$\lambda_D^2 \omega_{pe}^2 = v_{th}^2$$

where v_{th} is the electron thermal speed.