Basic Plasma Physics

Steve Schwartz

Imperial College London

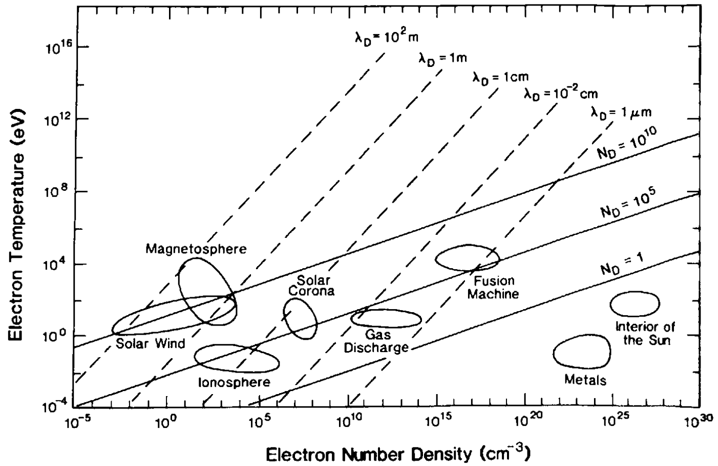
10 October 2011

Contents

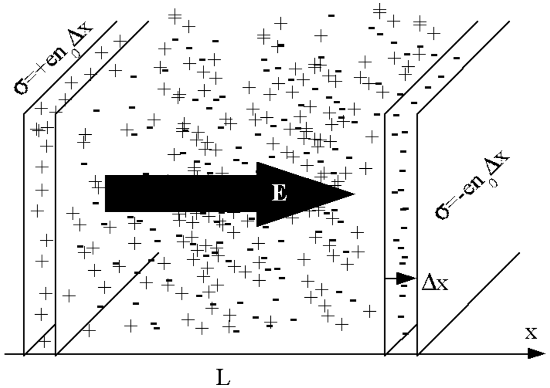
Basics - What is a plasma?

- Ionised gas in which dynamics dominated by EM
- Collective behaviour - Debye shielding
- Isolated excess charge $+Q$ attracts cloud of electrons that shield it
- $\phi = \frac{Q}{4\pi\epsilon_o r}$ for a point charge in vacuum
- $\phi = \frac{Q}{4\pi\epsilon_o r} e^{-r/\lambda_D}$ in plasma
- $\lambda_D = \sqrt{\frac{\epsilon_o k_b T}{e^2 n_o}}$
- Need $N_D \equiv n_o \lambda_D^3 \gg 1$ for shielding to work
- Plasmas are overall charge neutral on scale $\gg \lambda_D$

Some plasmas



Collective Behaviour - Plasma Oscillations



- Displacements give rise to restoring forces
- $\omega = \omega_{pe} = \sqrt{\frac{n_o e^2}{m_e \epsilon_o}}$
- Independent of wavelength (cf. $\omega = kc$) so $\mathbf{v}_{group} = 0$
- Most plasmas are collisionless

Single Particle Motion

Despite the intrinsic collective nature, looking at individual particles is instructive, and also relevant for, e.g., energetic particles.
Lorentz force

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

Note $\mathbf{B}$ does no work on particle, since

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) \equiv m\mathbf{v} \cdot \frac{d\mathbf{v}}{dt} = \mathbf{v} \cdot q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) = q\mathbf{v} \cdot \mathbf{E}$$

Motion along $\mathbf{B}$ is simple:

$$m\frac{dv_\parallel}{dt} = qE_\parallel$$

Gyromotion

Magnetic force is $\perp$ to $\mathbf{v}$ and gives rise to circular motion
Speed is constant, so

$$\frac{mv_\perp^2}{R} = qv_\perp B$$

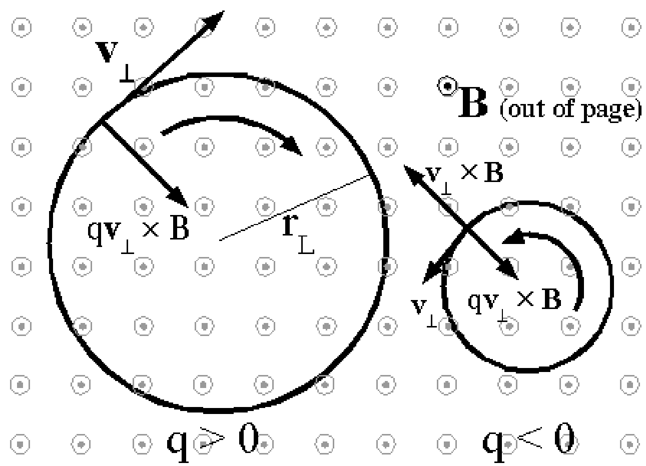
which gives the parameters of the (non-relativistic) cyclotron or gyromotion:

Larmor Radius $R_L = \frac{mv_\perp}{|q|B}$

Cyclotron Frequency $\Omega_c = \frac{qB}{m}$

Positive particles are left-handed

Gyromotion in action



Helical Motion

So the general motion of a particle in EM fields tends to be

- Motion along **B** (e.g., constant if $\mathbf{E} = \mathbf{0}$ or simple acceleration)
- Gyromotion around magnetic field
- $\Rightarrow$ helical motion

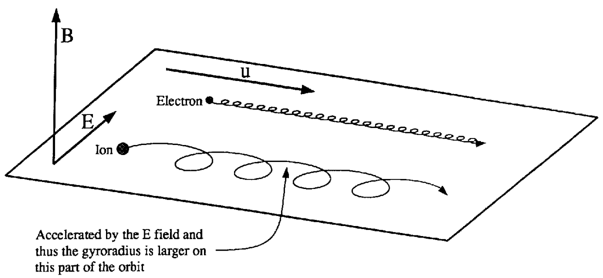
So particles tend to be tied to field lines (keep this thought)

Particle Drifts - $\mathbf{E} \times \mathbf{B}$

If we add $\mathbf{E}_\perp$ to the problem, something very interesting happens:
ALL particles drift $\perp$ **B** at a velocity $\mathbf{E} \times \mathbf{B}/B^2$.
Essentially, if we move with this velocity the electric field transforms s.t. $\mathbf{E}_\perp \to \mathbf{0}$ so motion in this frame simple gyromotion.

Particle Drifts - $\mathbf{E} \times \mathbf{B}$

If we add $\mathbf{E}_\perp$ to the problem, something very interesting happens:
ALL particles drift $\perp$ **B** at a velocity $\mathbf{E} \times \mathbf{B}/B^2$.
Essentially, if we move with this velocity the electric field transforms s.t. $\mathbf{E}_\perp \to \mathbf{0}$ so motion in this frame simple gyromotion.



Can also see this qualitatively by looking at local R_L.
Note: Force due to **E** leads to drift $\perp$ to force!

Particle Drifts - Other forces

For a general force **F** on a particle, we can think of an effective electric field

$$\mathbf{E}_{eff} \equiv \mathbf{F}/q$$

and see immediately that this will lead to a drift

$$\mathbf{v}_F = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

that depends on charge.

Particle Drifts - Curvature and Gradient Drifts

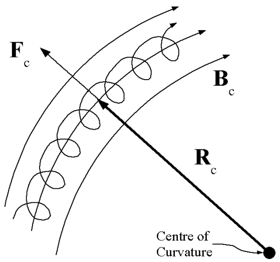
Curvature drift

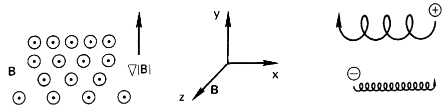
$$\mathbf{F}_c = \frac{mv_\parallel^2}{R_c}\hat{\mathbf{R}}_c$$

$$\mathbf{v_c} = \frac{\mathbf{F_c} \times \mathbf{B}}{qB^2}$$

Gradient drift

$$\mathbf{v}_{\nabla B} = \frac{mv_\perp^2}{2qB^3}(\mathbf{B} \times \nabla|\mathbf{B}|)$$





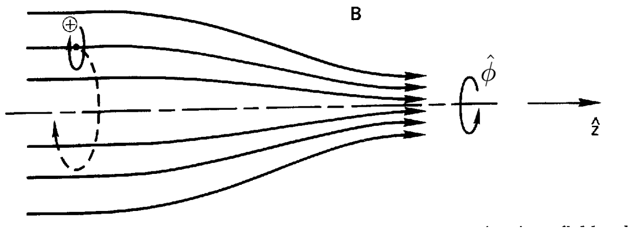
Useful Concepts

Pitch angle (= pitch of helix)	$\alpha = tan^{-1} v_\perp / v_\parallel$
Guiding Centre	(Motion) of centre of gyromotion
Magnetic moment (current × area)	$\mu_m = \frac{mv_\perp^2}{2B} = \frac{W_\perp}{B}$

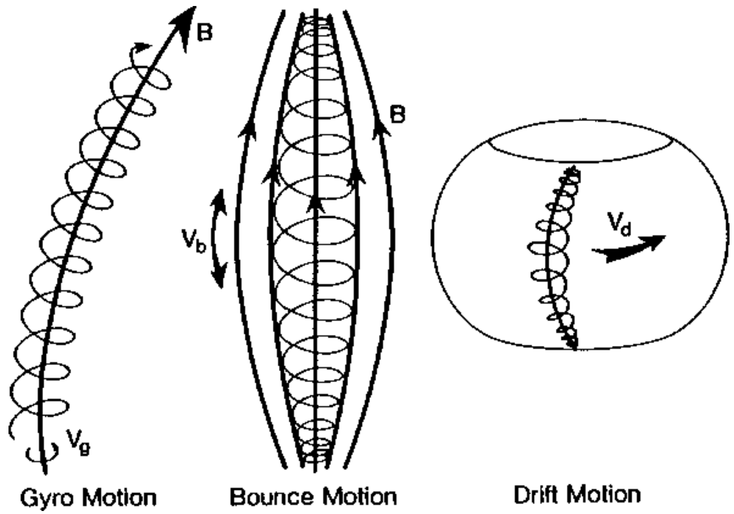
μ_m is an "Adiabatic Invariant;" it is a constant of the motion in slowly-varying ($L \gg R_L$, $T \gg 1/\Omega_c$) conditions.

Magnetic Mirroring

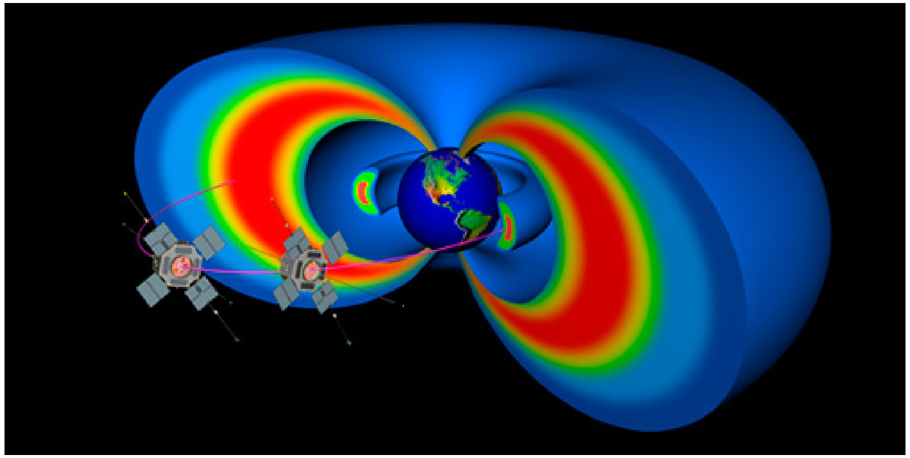
Since μ_m = constant, if $B \nearrow$ then $W_\perp \nearrow$.
B can't change $W = W_\parallel + W_\perp$, so $W_\parallel \searrow$.
If $W_\parallel \to 0$, the particle mirrors and can be trapped



Mirroring in the Magnetosphere



Van Allen Belts



Magnetohydrodynamics - MHD

Basic concepts:

- Treat plasma as fluid (this is simplest collective description)
- Equation of motion of a fluid element
- Overall charge neutral $\Rightarrow$ no electric forces
- Need equation of state
- Need an Ohm's Law
- Maxwell's Equations to describe EM fields $\Rightarrow$ coupled problem
- If need to know more fine detail, treat as multi-fluid or kinetically
- Can *derive* fluid description from kinetic theory; need closure assumption

MHD - Fluid equations

Mass conservation (continuity), Newton's Law, Equation of State:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \quad (1)$$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \mathbf{V} = -\nabla p + \mathbf{j} \times \mathbf{B} \quad (2)$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) (p\rho^{-\gamma}) = 0 \quad (3)$$

$$(4)$$

The operator

$$\frac{d}{dt} \equiv \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \quad (5)$$

measures rate of change following an element of fluid.

MHD - Maxwell's Equations

- With charge neutrality, don't need Gauss's Law
- $\nabla \cdot \mathbf{B} = 0$ will hold, but is just an initial condition on induction equation
- Ignore displacement current (non-relativistic):

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (6)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \quad (7)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (8)$$

- Ohm's Law for moving conductor:

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \frac{\mathbf{j}}{\sigma}$$

Induction

Manipulating Faraday's Law and Ohm's Law leads eventually to the MHD Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

- RHS represents convection and diffusion of **B**
- Magnetic Reynold's Number

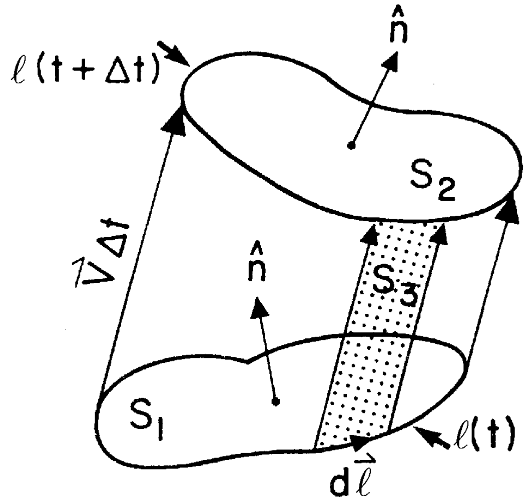
$$\frac{\text{convection}}{\text{diffusion}} \equiv R_m \equiv \frac{VB/L}{B/\mu_0 \sigma L^2} = \mu_0 \sigma V L$$

- $R_m \ll 1$ (thin layers, poor conductors) field diffuses and dissipates

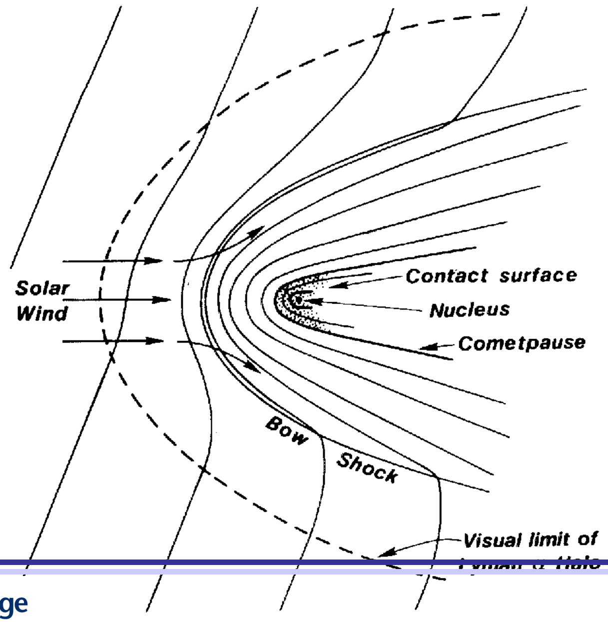
$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mu_o \sigma} \nabla^2 \mathbf{B}$$

Flux Freezing

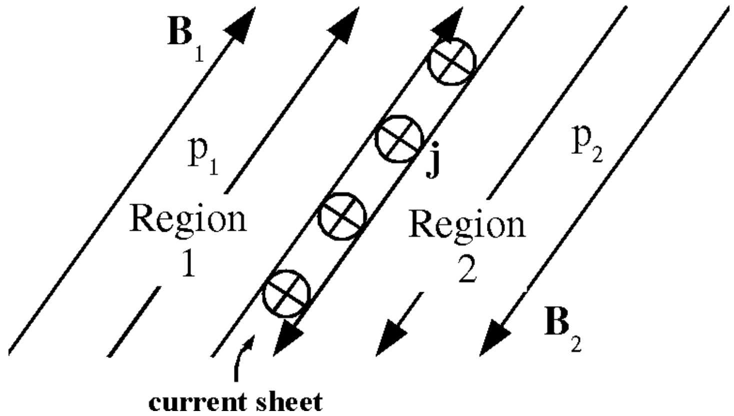
$R_m \gg 1$ plasma and magnetic flux are "frozen" to one another



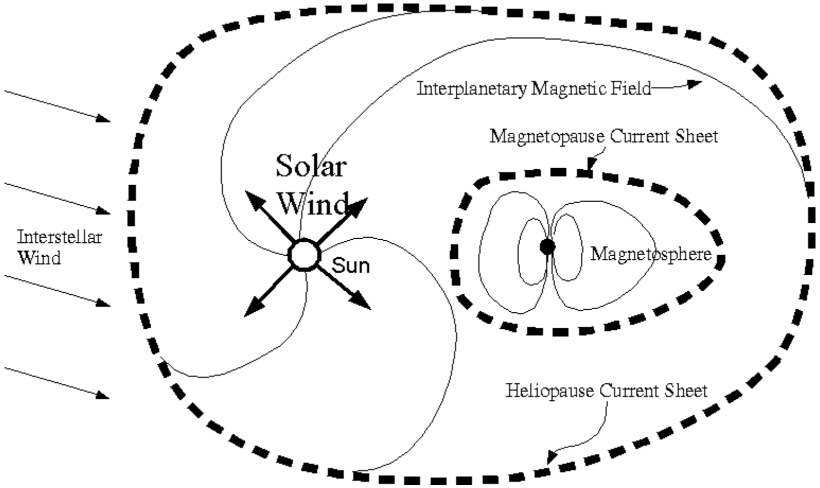
Applications of Flux Freezing - Comets



Applications of Flux Freezing - Current Sheets and Boundary Layers

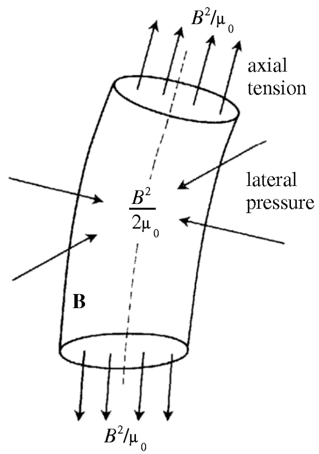


Applications of Flux Freezing - Cell Model of Plasma Universe



Magnetic Forces

- Substitute for $\mathbf{j}$ from Ampere's Law
- A lot of manipulation
- $\mathbf{j} \times \mathbf{B} = -\nabla_{\perp}\left(\frac{B^2}{2\mu_0}\right) - \frac{B^2}{\mu_0}\frac{\mathbf{R}_c}{R_c^2}$
- $\Rightarrow$ magnetic forces act like a (2D) pressure in directions perpendicular to $\mathbf{B}$
- Plus tension (straightening curves) along $\mathbf{B}$.
- Equilbria are balance of magnetic and thermal pressure gradient forces
- Not all equilibria are stable!

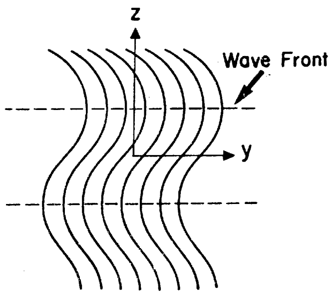


MHD Waves

- Waves illustrate forces in a medium
- Waves transport energy and momentum
- Waves can accelerate (or decelerate) energetic particles
- Waves are fun

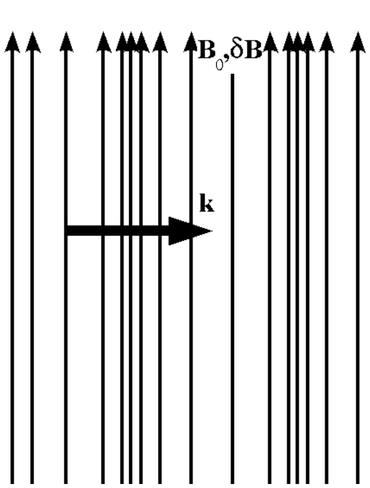
Alfvén Waves

- "Magnetic Tension" suggests waves on a string
- Speed of propagation $v_A = \sqrt{T/\rho} = \sqrt{B^2/\mu_o\rho}$
- $\omega = \pm k_{\parallel} v_A$
- NB $\mathbf{v}_{group} = v_A \hat{\mathbf{B}}_o$
- $\frac{\delta \mathbf{V}}{v_A} = \pm \frac{\delta \mathbf{B}}{B_o}$
- $\delta\rho = 0$

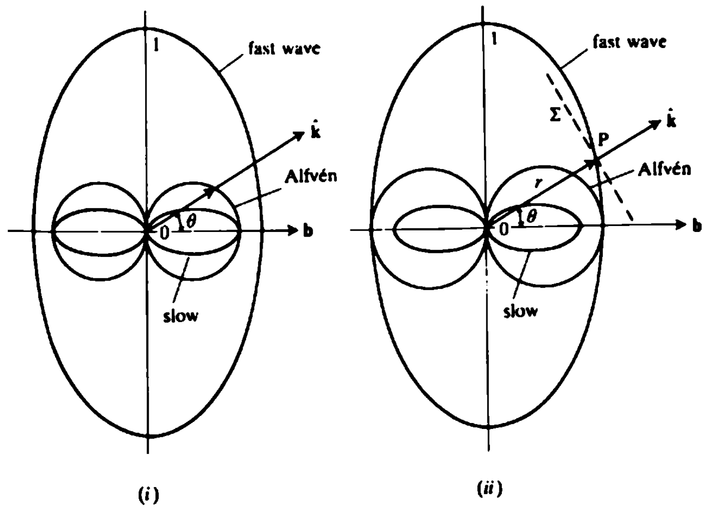


Magnetosonic Waves

- "Magnetic Pressure" can work with thermal pressure
- Speed of propagation something like $v_{ms} = \sqrt{\text{“}\gamma \mathcal{P}\text{“}/\rho}$
- Here "$\gamma \mathcal{P}$" $= \gamma P_{thermal} + 2P_{magnetic}$
- 2 because B pressure is 2D
- So $v_{ms}^2 = (\gamma P/\rho) + (2B^2/2\mu_o\rho)$
- i.e., $v_{ms}^2 = c_s^2 + v_A^2$
- Above ok for $\mathbf{k} \perp \mathbf{B}_o$
- Can also get slow mode, in which thermal and magnetic compete



General Propagation



$v_A < c_s$ $v_A > c_s$

What have we left out?

- Kinetics
  - Collisions
  - Non-thermal particles & acceleration
  - Microinstabilities & anomalous transport
- Discontinuities (Shocks, TD's, RD's)
- Dynamos
- Relativity
- Magnetic Reconnection
- Real plasmas
  - Solar wind, magnetosphere
  - Solar atmosphere
  - Partially ionised media (ionosphere)
- Multi-fluid
- Simulations, data analysis techniques
- and a whole lot more!