Post-graduate Lectures in Basic Plasmas
Space & Atmospheric Physics
Imperial College London
Problems
(from online book http://www.sp.ph.ic.ac.uk/ sjs/APmaster.pdf)
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Chapter 2: Basic Concepts

1 The solar wind at 1AU is a fully ionised proton-electron plasma with density $\sim$ 5 particles/cm^3 and an electron energy $\sim$ 3eV. Calculate the electron temperature in Kelvin, the Debye length λ_d, Debye number N_D, and plasma frequency ω_{pe}. [Be careful with units. We've used traditional units rather than SI ones here. 1eV $\equiv$ 1 electron Volt = $1.6e^{-19}$ Joules.]

2 A hydrogen plasma consists of protons and electrons with densities $n_p = n_e = n_0$. The electrons can be considered to be free to move while the ions are considered fixed on a uniform grid due to their relatively slower thermal motion. A point test charge q is placed in the plasma and establishes a resulting potential ϕ, the charged shielded Coulomb potential given in Equation 2.25

$$\phi = \frac{A}{r} e^{-r/\lambda_d}$$

If the electron population has temperature T, they can be expected to be distributed in a potential according to the Boltzmann factor $n_e(r) = n_0 e^{-(-e\phi/k_b T)}$. Assuming $|e\phi/k_b T| \ll 1$, expand this expression for $n_e(r)$ to find an expression for the net charge density $\rho_q(r) = en_p - en_e(r)$ and use this to show that the potential $\phi(r)$ is consistent with Poisson's equation $\nabla^2\phi = -\rho_q/\epsilon_0$. (Use the spherical polar form of the Laplacian $\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\phi}{\partial\phi^2}$ with spherical symmetry.)

Chapter 3: Individual Particle Motion

1 Calculate the Larmor radius and gyrofrequencies for the following particles

1. A 10 keV electron moving with a pitch angle of 45° with respect to the Earth's magnetic field of 30,000nT [nanoTeslas].
2. A solar wind proton moving at 400 km/s perpendicular to a field of 5nT.
3. A 1 keV He^+ ion in the solar atmosphere near a sunspot where $B = 5 \times 10^{-2}$T ($v_{||} = 0$).

2 The magnetic field strength in the Earth's magnetic equatorial plane is given by

$$B = B_0\left(\frac{R_e}{r}\right)^3$$

where $B_0 = 0.3 \times 10^{-4}$T, R_e is the Earth radius ($\sim$ 6400km), and r is the geocentric distance. Derive an expression for the drift period (the time for 1 orbit of the Earth) of the particle under the influence of the ∇B drift. Evaluate this period for both a proton and an electron of 1keV at a distance $5R_e$ from the Earth. Compare the answer to (a) the drift induced by the force of gravity on the same particles and (b) the orbital period of an uncharged particle at the same position. (The mass of the Earth is 6×10^{24}kg).

3 (a bit harder) A non-relativistic particle of mass m and charge q moves in a steady, axially symmetric magnetic field of a sunspot, which is taken to be uniform below the solar surface and falls off with height, h, above the surface as

$$B = B_0 \frac{H^3}{H^3 + h^3}$$

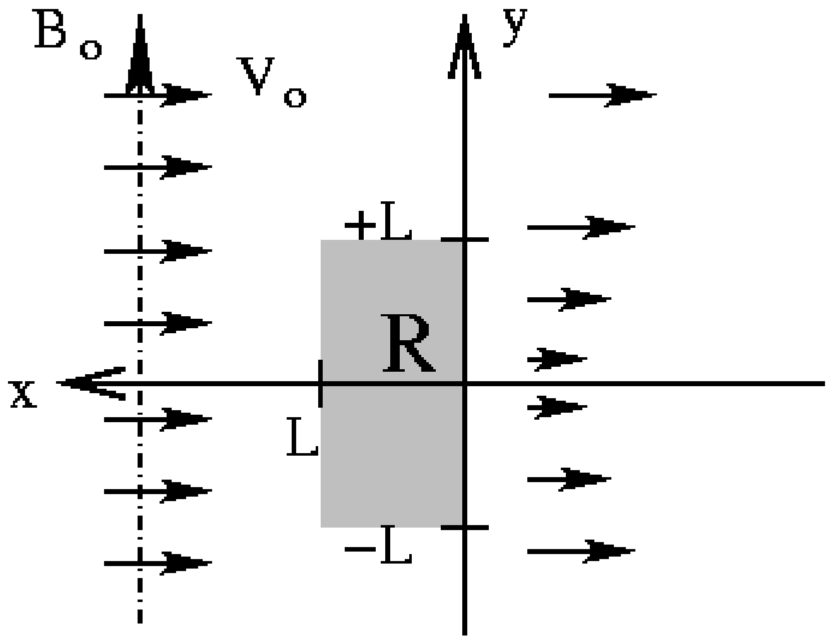

Figure 1: Flow hitting an obstacle.

for $h > 0$, where H is a constant and B_0 is the value of the field at and below the surface. [In effect, the field is roughly dipolar for large h].

1. Sketch the field lines in this model of a sunspot field
2. If particles of speed v are produced at the surface with an isotropic distribution (i.e., they move equally likely in all directions), and conserve their magnetic moments μ_m, show that as they move to successively greater heights h the particles possess a smaller range of pitch angles, $\alpha \equiv \arccos(v_{\|}/v)$. [Hint: Draw a picture of a spherical shell in $(v_{\|}, v_{\perp})$ space, and consider those particles with pitch angles near 90°.]
3. Hence explain how this process of "magnetic focussing" produces a "beam-like" particle distribution. How narrow is this beam in angular spread (i.e., pitch angle) at $h \sim 2H$?

Chapter 4: MagnetoHydroDynamics

1 A perfectly conducting plasma has a velocity $-V_0\hat{\mathbf{x}}$ and is threaded by a magnetic field $B_0\hat{\mathbf{y}}$. An obstacle stands in the flow, and causes a deceleration of the plasma within the region R such that

$$\frac{dV}{dx} = -\left(1 - \left|\frac{y}{L}\right|\right) 0.9 \frac{V_0}{L}$$

Assume that the flow field is fixed once the plasma moves into the region $x < 0$, i.e., that $V(x < 0, y) = V(x = 0, y)$, and that the flow remains in the negative x-direction (i.e., there is no deflection of the flow by the obstacle). Take the $\hat{\mathbf{z}}$ direction to be invariant. (see Figure 1)

1. By applying your knowledge of the frozen-in flux principle, sketch the magnetic field structure in the $x - y$ plane.
2. What is the magnetic field strength at $(0, 0)$?
3. What has happened to the kinetic energy of the flow passing through the region R? (How does conservation of energy apply?)
4. What would really happen in the region $x < 0$ and why?

3 Consider a uniform plasma of density ρ threaded by a uniform magnetic field **B**. By making an analogy to the case of small perturbations on a string in which the mass per unit length is ρ and the tension T is equivalent to the magnetic field tension B^2/μ_0, show that the transverse waves on the string propagate with a phase velocity equivalent to the Alfvén speed.

6 Consider the equilibrium of an isolated cylindrically symmetric flux tube of radius r_0 immersed in a uniform field $B_z\hat{\mathbf{z}}$. Given that the plasma thermal pressure varies as $p(r) = p_0\left(1 - \frac{r}{r_0}\right)$ show that the azimuthal magnetic field must vary according to

$$\frac{B_\phi^2}{2\mu_0} = \frac{r}{3r_0}p_0 \qquad (1)$$

[You will need to solve Equation 4.68 or at least show that this is a valid solution in the region $r \leq r_0$. You will also need to assign appropriate boundary conditions at, say, $r = 0$.] Also, find the electric current density $\mathbf{j} = \nabla \times \mathbf{B}/\mu_0$ for this configuration.